International Journal of Trend in Scientific Research and Development (IJTSRD)
International Open Access Journal
ISSN No: 2456 - 6470 | www.ijtsrd.com | Volume - 2 | Issue – 5

Integral Root Labeling of $P_m \cup G$ Graphs

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ABSTRACT
Let $G = (V, E)$ be a graph with p vertices and q edges. Let $f: V \to \{1,2, \dots q+1\}$ is called an **Integral Root labeling** if it is possible to label all the vertices $v \in V$ with distinct elements from $\{1,2, \dots q+1\}$ such that it induces an edge labeling $f^+: E \to \{1,2, \dots q\}$ defined as

$f^+(uv) = \left\lceil \sqrt{\frac{(f(u))^2+(f(v))^2+f(u)f(v)}{3}} \right\rceil$ is distinct for all $uv \in E$. (i.e.) The distinct vertex labeling induces a distinct edge labeling on the graph. The graph which admits Integral Root labeling is called an **Integral Root Graph**.

In this paper, we investigate the Integral Root labeling of $P_m \cup G$ graphs like $P_m \cup P_n$, $P_m \cup (P_n \odot K_1)$, $P_m \cup L_n$, $P_m \cup (P_n \odot K_{1,2})$, $P_m \cup (P_n \odot K_{1,3})$, $P_m \cup (P_n \odot K_1) \odot K_{1,2}$

Key words: *P_m∪P_n, P_m∪(P_n ⊙K_1), P_m∪L_n, P_m∪(P_n ⊙K_1,2), P_m∪(P_n ⊙K_1,3), P_m∪(T_n ⊙K_1), P_m∪(P_n ⊙K_1)⊙K_1,2*

INTRODUCTION

The graph considered here will be finite, undirected and simple. The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$. For all detailed survey of graph labeling we refer to Gallian [1]. For all standard terminology and notations we follow Haray [2]. V.L Stella Arputha Mary and N.Nanthini introduced the concept of Integral Root Labeling of graphs in [8]. In this paper we investigate Integral Root labeling of $P_m \cup G$ graphs. The definitions and other informations which are useful for the present investigation are given below.

BASIC DEFINITIONS

Definition: 3.1
A walk in which $u_1, u_2, \dots u_n$ are distinct is called a **Path**. A path on n vertices is denoted by P_n

Definition: 3.2
The graph obtained by joining a single pendent edge to each vertex of a path is called a **Comb.**

Definition: 3.3
The Cartesian product of two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is a graph $G=(V,E)$ with $V=V_1 \times V_2$ and two vertices $u=(u_1 u_2)$ and $v=(v_1 v_2)$ are adjacent in $G_1 \times G_2$ whenever ($u_1=v_1$ and u_2 is adjacent to v_2) or ($u_2=v_2$ and u_1 is adjacent to v_1) .It is denoted by $G_1 \times G_2$.

Definition: 3.4
The Corona of two graphs G_1 and G_2 is the graph $G=G_1 \odot G_2$ formed by taking one copy of G_1 and $|(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition: 3.5
The product graph $P_2 \times P_n$ is called a **Ladder** and it is denoted by L_n

Definition: 3.6
The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $E = E_1 \cup E_2$.

Definition: 3.7
The graph $P_n \Theta K_{1,2}$ is obtained by attaching $K_{1,2}$ to each vertex of P_n.

Definition: 3.8
The graph $P_n \Theta K_{1,3}$ is obtained by attaching $K_{1,3}$ to each vertex of P_n.

Definition: 3.9
A graph that is not connected is disconnected. A graph G is said to be disconnected if there exist two nodes in G such that no path in G has those nodes as endpoints. A graph with just one vertex is connected. An edgeless graph with two (or) more vertices is disconnected

MAIN RESULTS
Theorem: 4.1
$P_m \cup P_n$ is an Integral Root graph.

Proof:
Let $P_m = u_1, u_2, \ldots, u_m$ be a path on m vertices.
Let $P_n = v_1, v_2, \ldots, v_n$ be another one path on n vertices.
Let $G = P_m \cup P_n$.
Define a function $f: V(G) \to \{1,2,\ldots,q+1\}$ by

$$f(u_i) = i; \qquad 1 \leq i \leq m;$$
$$f(v_i) = m + i; \quad 1 \leq i \leq n.$$

Then we find the edge labels

$$f^+(u_i u_{i+1}) = i; \qquad 1 \leq i \leq m-1;$$
$$f^+(v_i v_{i+1}) = m + i; \quad 1 \leq i \leq n-1.$$

Then the edge labels are distinct.
Hence $P_m \cup P_n$ is a Integral Root graph.

Example: 4.2

An Integral Root labeling of $P_5 \cup P_6$ is show below.

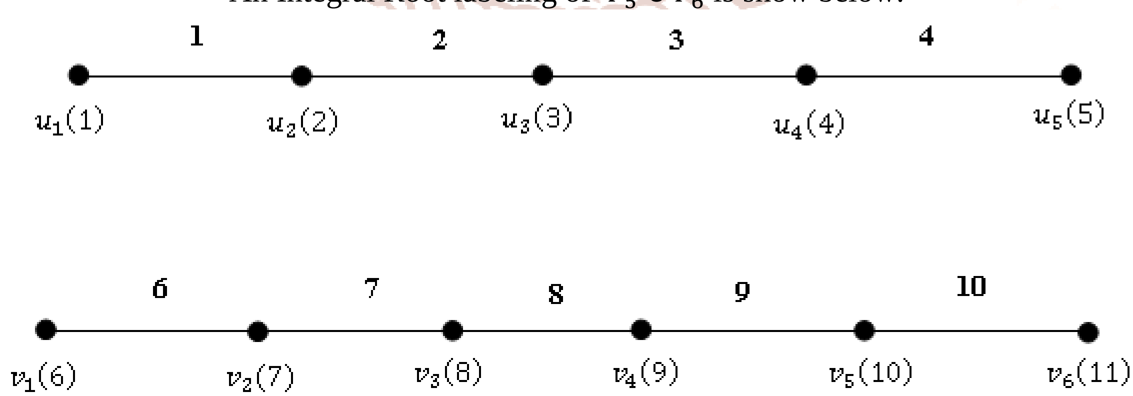

Figure: 1

Theorem: 4.3
$P_m \cup (P_n \Theta K_1)$ is a Integral Root graph.

Proof:

Let $P_n \Theta K_1$ be a Comb graph obtained from a path $P_n = v_1, v_2, \ldots., v_n$ by joining a vertex u_i to v_i, $1 \le i \le n$.
Let $P_m = w_1, w_2, \ldots, w_m$ be a path.
Let $G = P_m \cup (P_n \Theta K_1)$.
Define a function $f: V(G) \to \{1,2, \ldots., q+1\}$ by

$$f(w_i) = i; \quad 1 \le i \le m;$$
$$f(v_i) = m + 2i - 1; \quad 1 \le i \le n;$$
$$f(u_i) = m + 2i; \quad 1 \le i \le n.$$

Then we find the edge labels are

$$f^+(w_i w_{i+1}) = i; \quad 1 \le i \le m-1;$$
$$f^+(v_i v_{i+1}) = m + 2i; \quad 1 \le i \le n-1;$$
$$f^+(v_i u_i) = m + 2i - 1; \quad 1 \le i \le n-1.$$

Then the edge labels are distinct.
Hence $P_m \cup (P_n \Theta K_1)$ is an Integral Root graph.

Example: 4.4

An Integral Root labeling of $P_5 \cup (P_5 \Theta K_1)$ is given below.

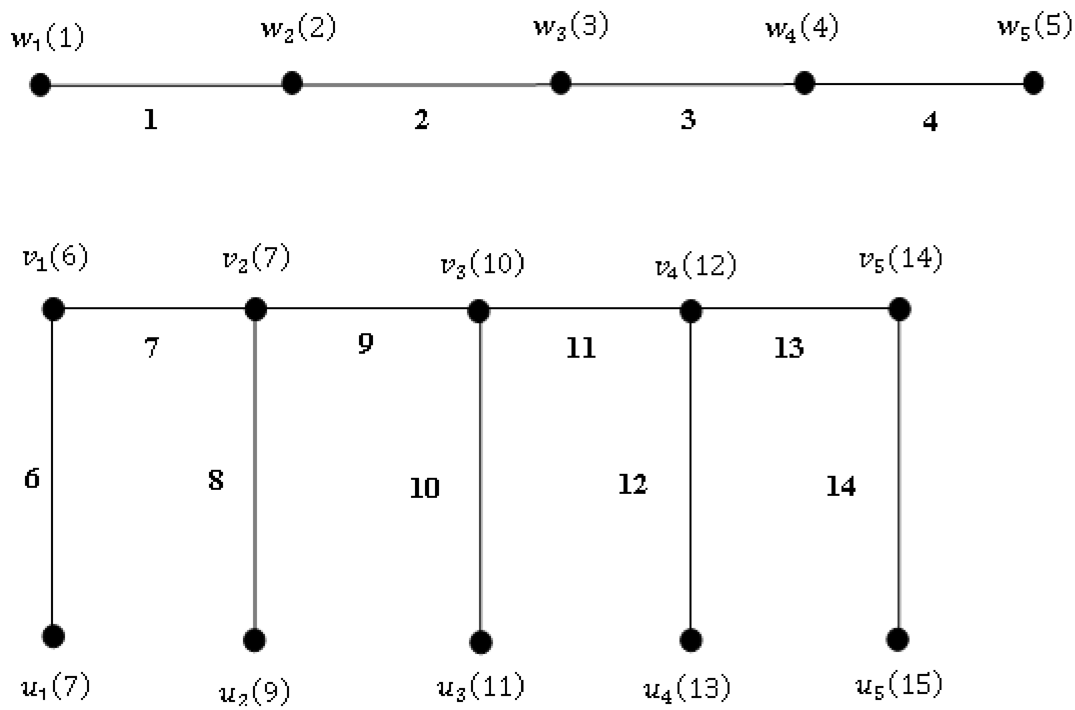

Figure: 2

Theorem: 4.5

$P_m \cup L_n$ is an Integral Root graph.

Proof:

Let $P_m = u_1, u_2, \ldots., u_m$ be a path.
Let $\{v_1, v_2, \ldots, v_n, w_1, w_2, \ldots\ldots, w_n\}$ be the vertices of ladder.
efine a function $f: V(G) \to \{1,2, \ldots., q+1\}$ by

$$f(u_i) = i; \quad 1 \le i \le m;$$
$$f(v_i) = m + 3i - 2; \quad 1 \le i \le n;$$
$$f(w_i) = m + 3i - 1; \quad 1 \le i \le n.$$

Then we find the edge labels

$$f^+(u_iu_{i+1}) = i; \qquad 1 \le i \le m-1;$$
$$f^+(v_iv_{i+1}) = m + 3i - 1; \quad 1 \le i \le n-1;$$
$$f^+(w_iw_{i+1}) = m + 3i; \qquad 1 \le i \le n-1.$$
$$f^+(v_iw_i) = m + 3 - 2i; \qquad 1 \le i \le n.$$

Then the edge labels are distinct.
Hence $P_m \cup L_n$ is an Integral Root graph.

Example: 4.6

An Integral Root labeling of $P_5 \cup L_5$ is displayed below.

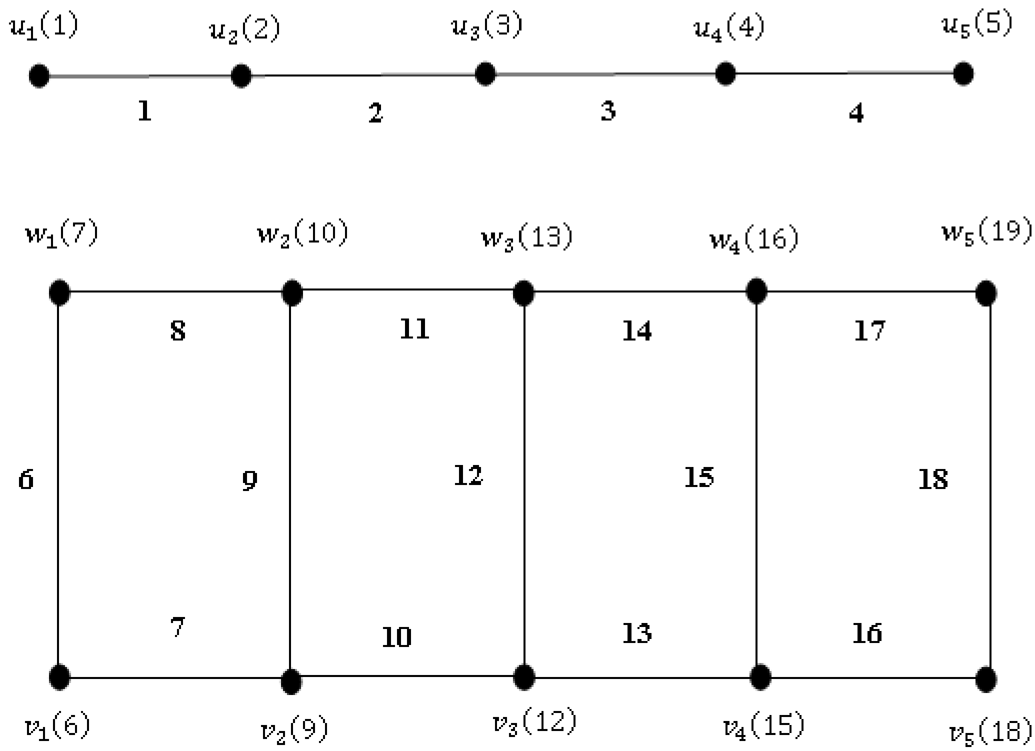

Figure: 3

Theorem: 4.7

$P_m \cup (P_n \Theta K_{1,2})$ is a Integral Root graph.

Proof:

Let $P_n \Theta K_{1,2}$ be a graph obtained by attaching each vertex of a path P_n to the central vertex of $K_{1,2}$.
Let $P_n = v_1, v_2, \dots, v_n$ be a path.
Let w_i and x_i be the vertices of $K_{1,2}$ which are attaching with the vertex v_i of P_n
$1 \le i \le n$.

Let $P_m = u_1, u_2, \dots, u_m$ be a path.
Let $G = P_m \cup (P_n \Theta K_{1,2})$.

Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$$f(u_i) = i; \qquad 1 \le i \le m;$$
$$f(v_i) = m + 3i - 1; \quad 1 \le i \le n;$$
$$f(w_i) = m + 3i - 2; \quad 1 \le i \le n;$$
$$f(x_i) = m + 3i; \qquad 1 \le i \le n.$$

Then we find the edge labels are

$$f^+(u_i u_{i+1}) = i; \qquad 1 \le i \le m-1;$$
$$f^+(v_i v_{i+1}) = m + 3i; \qquad 1 \le i \le n-1;$$
$$f^+(v_i w_i) = m + 3i - 2; \quad 1 \le i \le n;$$
$$f^+(x_i v_i) = m + 3i - 1; \quad 1 \le i \le n.$$

Hence $P_m \cup (P_n \odot K_{1,2})$ is a Integral Root graph.

Example: 4.8

An Integral Root labeling of $P_5 \cup (P_3 \odot K_{1,2})$ is given below.

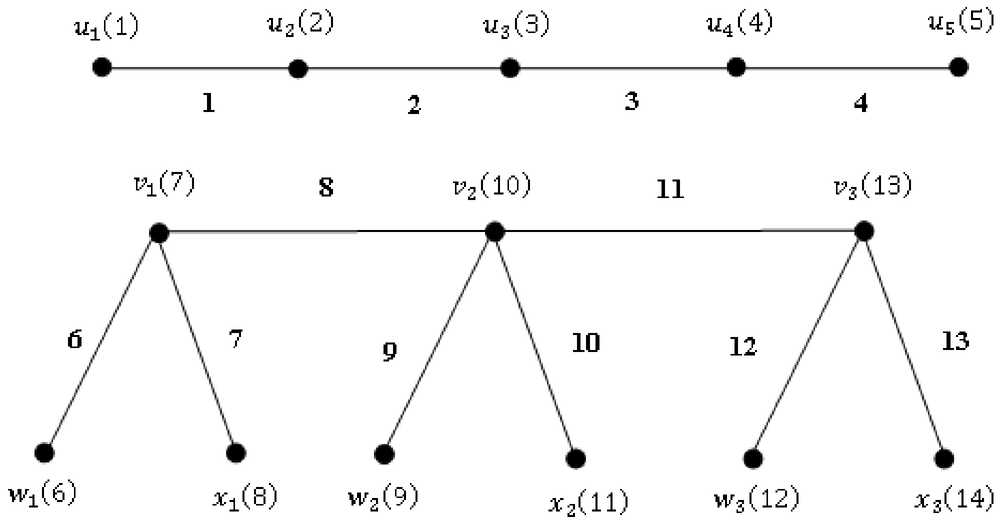

Figure: 4

Theorem: 4.9

$P_m \cup (P_n \odot K_{1,3})$ is a Integral Root graph.

Proof:

Let $P_n \odot K_{1,2}$ be a graph obtained by attaching each vertex of a path P_n to the central vertex of $K_{1,3}$.
Let $P_n = w_1, w_2, \dots, w_n$ be a path.
Let v_i, x_i and y_i be the vertices of $K_{1,3}$ which are attaching with the vertex w_i of P_n
$1 \le i \le n$.
Let $P_m = u_1, u_2, \dots, u_m$ be a path.
Let $G = P_m \cup (P_n \odot K_{1,3})$.

Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$$f(u_i) = i; \qquad 1 \le i \le m;$$
$$f(v_i) = m + 4i - 3; \quad 1 \le i \le n;$$
$$f(w_i) = m + 4i - 2; \quad 1 \le i \le n;$$
$$f(x_i) = m + 4i - 1; \quad 1 \le i \le n;$$
$$f(y_i) = m + 4i; \qquad 1 \le i \le n.$$

Then we find the edge labels are

$$f^+(u_i u_{i+1}) = i; \qquad 1 \le i \le m-1;$$
$$f^+(w_i w_{i+1}) = m + 4i; \quad 1 \le i \le n-1;$$
$$f^+(v_i w_i) = m + 4i - 3; \quad 1 \le i \le n;$$
$$f^+(x_i w_i) = m + 4i - 2; \quad 1 \le i \le n;$$
$$f^+(y_i w_i) = m + 4i - 1; \quad 1 \le i \le n.$$

Then the edge labels are distinct.
Hence $P_m \cup (P_n \odot K_{1,3})$ is a Integral Root graph.

Example: 4.10

An Integral Root labeling of $P_5 \cup (P_3 \Theta K_{1,3})$ is given below.

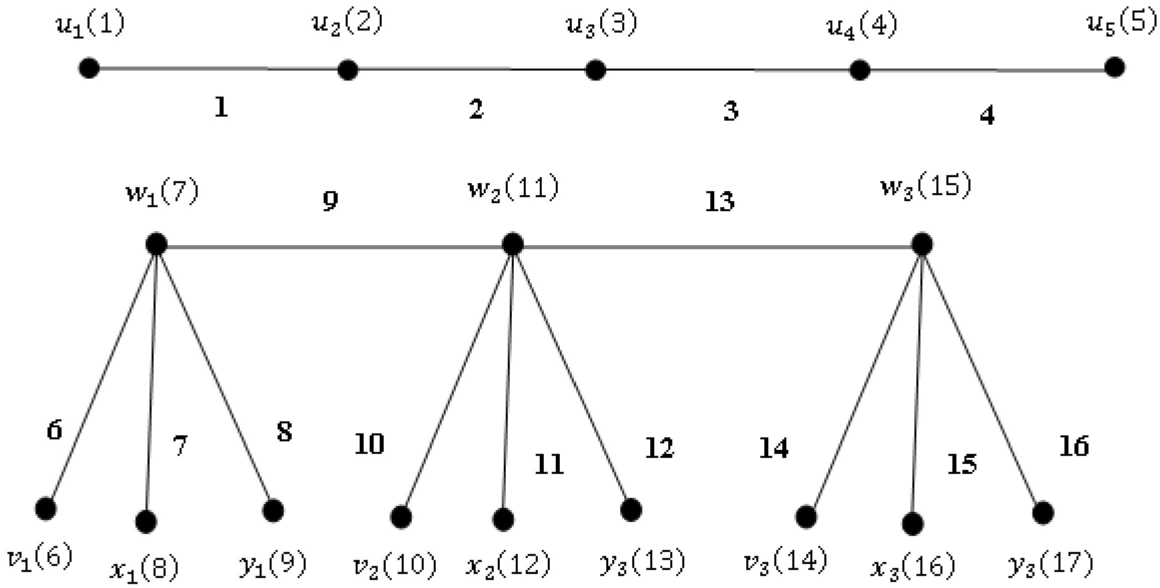

Figure: 5

Theorem: 4.11

$P_m \cup (P_n \Theta K_1) \Theta K_{1,2}$ is a Integral root graph.

Proof:

Let $G = P_m \cup (P_n \Theta K_1) \Theta K_{1,2}$.
Let $P_m = u_1, u_2, \ldots, u_m$ be a path.
Let G_2 be a comb and G_1 be the obtained by attaching $K_{1,2}$ at each pendant vertex of G_2.
Let its vertices be $v_i, w_i, x_i, y_i \quad 1 \leq i \leq n$.

Define a function $f: V(G) \to \{1,2, \ldots, q+1\}$ by

$$f(u_i) = i; \quad 1 \leq i \leq m;$$
$$f(v_i) = m + 5i - 3; \quad 1 \leq i \leq n;$$
$$f(w_i) = m + 5i - 4; \quad 1 \leq i \leq n;$$
$$f(x_i) = m + 5i - 2; \quad 1 \leq i \leq n;$$
$$f(y_i) = m + 5i; \quad 1 \leq i \leq n.$$

Then we find edge labels are

$$f^+(u_{i+1}u_i) = i; \quad 1 \leq i \leq m-1;$$
$$f^+(v_{i+1}v_i) = m + 5i - 1; \; 1 \leq i \leq n-1;$$
$$f^+(v_i w_i) = m + 5i - 4; \quad 1 \leq i \leq n;$$
$$f^+(w_i x_i) = m + 5i - 3; \quad 1 \leq i \leq n;$$
$$f^+(w_i y_i) = m + 5i - 2; \quad 1 \leq i \leq n.$$

Then the edge labels are distinct.
Hence $P_m \cup (P_n \Theta K_1) \Theta K_{1,2}$ is an Integral Root graph.

Example: 4.12

An Integral Root labeling of $P_5 \cup (P_4 \Theta K_1) \Theta K_{1,2}$ is given below.

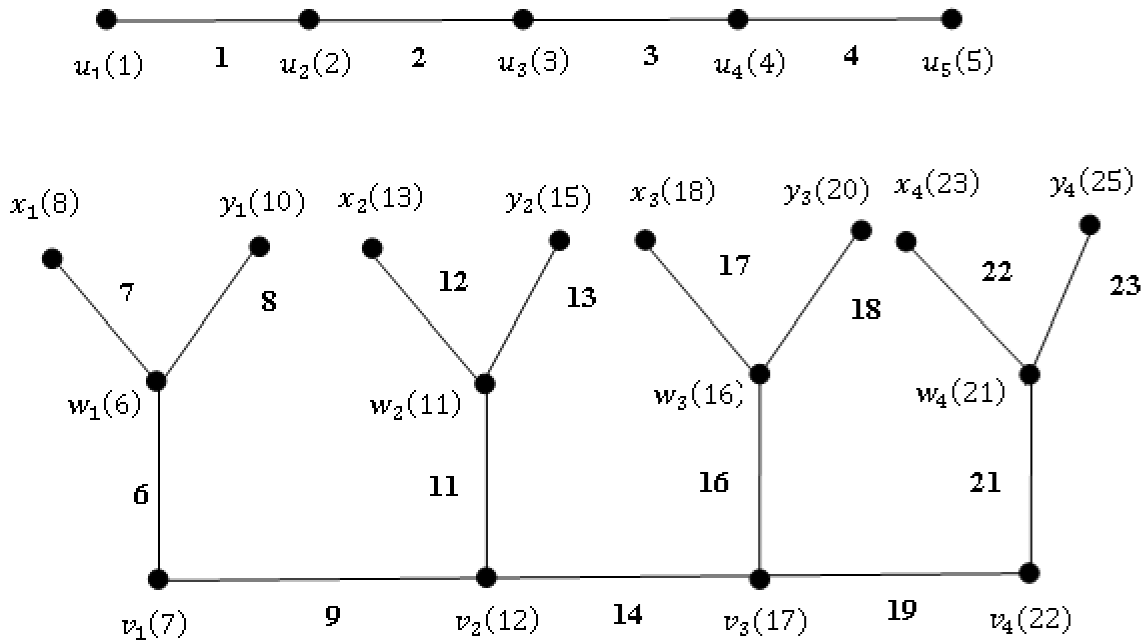

Figure: 6

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