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Rp-106: Formulation of Solutions of a Class of Standard Congruence of Composite Modulus of Higher Degree

Prof B M Roy

M. Sc (Maths), Hon. Ph. D, Hon. D. Sc, Head, Department of Mathematics,
Jagat Arts, Commerce & I H P Science College, Goregaon, Gondia, Maharashtra, India

ABSTRACT

In this paper, a standard congruence of higher degree of composite modulus is considered for study and after a thorough study, it is then formulated for its incongruent solutions. The established formula is tested and verified true by solving suitable examples. Formulation is the merit of the paper. This formulation made the study of the congruence of higher degree easy, simple and interesting. Mental calculation of solutions become possible due to this formulation. In the literature of earlier mathematics, no formulation is found for the solutions of the said congruence.

KEYWORDS: *Congruence of Higher Degree, Composite modulus, Formulation*

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INRODUCTION

Many research papers on the formulation of quadratic, cubic, bi-quadratic and congruence of higher degree of prime and composite modulus have been published in different reputed international journals by the author. Even the author found a special type of congruence of higher degree of composite modulus, yet not formulated. It can be stated as:

$$x^n \equiv a^n \ (mod\ a^m); a \neq 0, m > n".$$

The author wishes to consider such congruence and tried to find a formulation of the congruence of composite modulus of higher degree.

PROBLEM-STATEMENT

Here the problem is-

"To find a formulation of the congruence of higher degree of composite modulus of the type:

$$x^n \equiv a^n \ (mod\ a^m); a \neq 0, m > n \ in\ two\ cases:$$

Case-I: when n is even positive integer,
Case-II: when n is odd positive integer".

LITERATURE-REVIEW

Many books of Number theory containing congruence of higher degree are referred [1], [2]. No formulation of the said congruence is found. Only the research papers on the congruence of higher degree of the author published are seen [3], [4],……. …….[9].

ANALYSIS & RESULT

Case-I: Let n be an even positive integer and consider the congruence under consideration:

$$x^n \equiv a^n \ (mod\ a^m);\ m > n.$$

Also consider that $x \equiv a^{m-n+1}k \pm a \ (mod\ a^m)$

Then, $x^n \equiv (a^{m-n+1}k \pm a)^{\ n}$ (mod a^m)

$\equiv$
$(a^{m-n+1}k)^n \ n(\ a^{m-n+1}k)^{n-1}.a \ \frac{n.(n-1)}{2}\ (a^{m-n+1}k)^{n-2}.a^2 \pm \cdots \dots \dots \dots . . \pm n.a^{m-n+1}k.a^{n-1} + a^n \ (mod\ a^m)$

$\equiv a^n + a^m(\dots \dots)\ (mod\ a^m)$

$\equiv a^n \ (mod\ a^m)$

Thus, it can be said that $x \equiv a^{m-n+1}k \pm a \ (mod\ a^m)$ is a solution of the congruence.
But, if one has the value $k = a^{n-1}$,
then $x \equiv a^{m-n+1}k \pm a \ (mod\ a^m)$ reduces to

$x \equiv a^{m-n+1}.a^{n-1} \pm a \ (mod\ a^m)$

$\equiv a^m \pm a \ (modm\ )a^m)$

$\equiv\ 0 \pm a \ (mod\ a^m)$

$\equiv\ \pm a \ (mod\ a^m)$,

which are the same solutions as for k=0. Similarly, it can also be seen that for next higher values of k, the corresponding solutions repeats as for k=1, 2,………………,$(a^{n-1}-1)$.

Therefore, the required solutions are given by
$x \equiv a^{m-n+1}k \pm a \ (mod\ a^m)$; k 0, 1, 2, … … … , $(a^{n-1}-1)$.

It is also seen that for a single value of k, the congruence has two solutions and here k has a^{n-1} values. Thus, total number of solutions are definitely $2a^{n-1}$ for even n.

Case-II: Let n be an odd positive integer and consider the congruence under consideration: $x^n \equiv a^n \ (mod\ a^m);\ m > n$.

Also consider that $x \equiv a^{m-n+1}k \pm a \ (mod\ a^m)$

Then, $x^n \equiv (a^{m-n+1}k \pm a)^{\,n}$ (mod a^m)

$\equiv \pm a^n + a^m(\dots\dots) \ (mod\ a^m)$

$\equiv \pm a^n \ (mod\ a^m)$

Thus, $x \equiv a^{m-n+1}k \pm a \ (mod\ a^m)$ cannot be the solutions.

But if $x \equiv a^{m-n+1}k + a \ (mod\ a^m)$,
then , $x^n \equiv a^n \ (mod\ a^m)$.

Thus, it can be said that $x \equiv a^{m-n+1}k + a \ (mod\ a^m)$ is a solution of the congruence.

But, if one has the value $k = a^{n-1}$,
then $x \equiv a^{m-n+1}k + a \ (mod\ a^m)$ reduces to
$x \equiv a^{m-n+1}.a^{n-1} + a \ (mod\ a^m)$

$\equiv a^m + a \ (mod\ a^m)$

$\equiv\ 0 + a \ (mod\ a^m)$

$\equiv\ a \ (mod\ a^m)$,

which is the same solution as for k=0. Similarly, it can also be seen that for next higher values of k, the corresponding solutions repeats as for k=1, 2,……..,$(a^{n-1}-1)$.

Therefore, the required solutions are given by
$x \equiv a^{m-n+1}k + a \ (mod\ a^m)$; k = 1, 2, … … . , $(a^{n-1}-1)$.

It is also seen that for a single value of k, the congruence has one solution and here k has a^{n-1} values. Thus, total number of solutions are definitely a^{n-1} for odd n.

If $m \leq n$, the solutions of the congruence are given by
$x^n \equiv a^n \ (mod\ a^n)\ which\ is\ equivalent\ to$
$x^n \equiv 0 \ (mod\ a^n)$

and the solutions are given by: $x \equiv at \ (mod\ a^n), t\ being\ an\ integer$.

ILLUSTRATIONS

Consider the congruence $x^6 \equiv 729 \ (mod\ 2187)$.
It can be written as $x^6 \equiv 3^6 \ (mod\ 3^7) with\ a = 3$.
It is of the type

$x^n \equiv a^n \ (mod\ a^m); m > n\ with\ even\ positive\ integer\ n.$

It has $2a^{n-1} = 2.3^{6-1} = 2.3^5 = 2.243 = 486\ solutions$ as n is even positive integer and the solutions are given by
$x \equiv a^{m-n+1}k \pm a \ (mod\ a^m); k = 0, 1, 2, \dots\dots (a^5-1)$.

$\equiv 3^{7-6+1}k \pm 3 \ (mod\ 3^7)$

$\equiv 3^2k \pm 3 \ (mod\ 3^7)$

$9k\ 3 \ (mod\ 2187); k\ 0, 1, 2, 3, 4, \dots ,10, \dots\dots, 100, \dots\dots, (243 - 1)$.

$\equiv 0 \pm 3; 9 \pm 3; 18 \pm 3; 27 \pm 3; 36 \pm 3; \dots; 90\ 3; \dots\dots; 900 \pm 3; \dots\dots 2178 \pm 3$.

$\equiv 3, 2184; 6, 12; 15,21; 24, 30; 33, 39; \dots\dots\dots; 87, 93; \dots\dots;$

$897, 903; \dots\dots; 2175, 2181 \ (mod\ 2187)$.

These are the required 486 solutions.

These solutions are verified and found true.

Consider the congruence $x^5 \equiv 243 \ (mod\ 729)$.
It can be written as: $x^5 \equiv 3^5 (mod 3^6)$.

It is of the type:
$x^n \equiv a^n (mod a^m); m > n\ with\ n\ odd\ positive\ integer.$

The congruence has only $a^{n-1} = 3^{5-1} = 3^4 = 81\ solutions.$
Solutions are given by
$x \equiv a^{m-n+1}k + a \ (mod\ a^m); k = 0, 1, 2, 3,4, \dots\dots . (a^{n-1} - 1.$

$\equiv 3^{6-4}k + 3 \ (mod\ 3^6); k = 0, 1, 2, 3, 4, \dots\dots\dots . 80.$

$\equiv 3^2k + 3 \ (mod\ 729)$

$\equiv 9k + 3 \ (mod\ 729)$

$\equiv 0 + 3; 9 + 3; 18 + 3; 27 + 3; 36 + 3; 45 + 3; 54 + 3;\ \dots\dots .711 + 3;$

$720 + 3 \ (mod\ 729)$.

$\equiv 3, 12, 21, 30, 39, 48, 57, \dots\dots\dots\dots 714, 723 \ (mod\ 729)$.

These are the required 81 solutions.
These solutions are verified and found true.
Consider one more example: It can be written as: $x^6 \equiv 3^6 (mod\ 3^6)$.

It can be written as: $x^6 \equiv 0 \ (mod\ 3^6)$.

Then the solutions are given by

$x \equiv at\ (mod\ a^n), t\ being\ an\ integer.$
$\equiv 3t\ (mod\ 3^6); t = 1, 2, 3, \dots \dots ..$
$\equiv 3, 6, 9, 12, 15, \dots \dots \dots\ (mod\ 729)$

COLCLUSION

Thus, it can be concluded that the standard congruence of composite modulus of higher degree of the type: $x^n \equiv a^n\ (mod\ a^m); m > n$, has $2a^{n-1}$

solutions, if n is an even positive integer, given by $x \equiv a^{m-n+1}k \pm a\ (mod\ a^m); k = 0, 1, 2, \dots .. (a^{n-1} - 1)$.

But the congruence has only a^{n-1} solutions, if n is an odd positive integer, given by
$x \equiv a^{m-n+1}k + a\ (mod\ a^m); k = 0, 1, 2, \dots .. (a^{n-1} - 1)$.

If $m \leq n$, the solutions of the congruence are given by $x^n \equiv a^n\ (mod\ a^n)$; and the solutions are given by: $x \equiv at\ (mod\ a^n), t\ being\ an\ integer$.

MERIT OF THE PAPER

In this paper, a standard congruence of composite modulus of higher degree is formulated successfully. The formula established is proved true using suitable examples. It is verified by illustrations. The solutions can be obtained in a short time. It is proved time-saving. Formulation is the merit of the paper.

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