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RP-170: Formulating Solutions of a Special Class of Standard Cubic Congruence Modulo p^2 Multiple of 3^n

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ABSTRACT

In this research paper, the author has formulated the solutions of a standard cubic congruence of composite modulus modulo p^2 multiple of 3^n, n being a positive integer. It is found that the said congruence has exactly 3p incongruent solutions; p being an odd prime positive integer. It seems that the earlier mathematicians didn't study such types of congruence. First time a formulation of solutions is established by the author. Formulation is the merit of the paper as formulation of solutions has provided a simple procedure of finding the required solutions with an ease.

KEYWORDS: *Cubic Congruence, Chinese Remainder Theorem, Formulation*

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INTRODUCTION

Congruence being a part of modular arithmetic which is an inseparable part of Number Theory is included in pure mathematics. It is a general observation that the standard quadratic congruence of prime modulus were studied in the universities but no discussion is found about standard cubic and standard bi-quadratic congruence. Nowhere found any discussion of it. Here in this paper, the author considered a standard cubic congruence of composite modulus of very special type. It is of the type: $x^3 \equiv p^3 (mod\ p^2.3^n)$, p being an odd prime positive integer.

PROBLEM-STATEMENT

Here the problem is-

"To find the formula for solutions of the cubic congruence: $x^3 \equiv p^3 (mod\ p^2.3^n)$, p an odd prime".

LITERATURE REVIEW

Nothing is found about the said cubic congruence in the literature of Mathematics. Even the Books of Number Theory said a little [1], [2], [3]. Only a definition of standard cubic congruence is seen in [1]; also a definition of cubic residues is seen in [2]. Butthough there are some demerits, Chinese Remainder Theorem can be used to find all the solutions of the said congruence by splitting the original congruence into individual congruence and solving separately.

NEED OF RESEARCH

The demerits of CRT make unpopular among the readers and they all hesitate to use the said procedure. They feel uncomfortable to use it in solving the congruence. Here lies the need of formulation of the solutions. Therefore, the author has formulated the solutions of the congruence. The author already has formulated some standard cubic congruence of composite and prime modulus [4], [5], [6], [7], [8], [9].

ANALYSIS & RESULTS

Consider the congruence under consideration: $x^3 \equiv p^3 (mod\ p^2.3^n)$.

For the solutions, let $x \equiv p^{2-1}.3^{n-1}k + p\ (mod\ p^2.3^n)$

$\equiv p.3^{n-1}k + p\ (mod\ p^2.3^n)$.

Then, $x^3 \equiv (p.3^{n-1}k + p\ )^3\ (mod\ p^2.3^n)$

$$\equiv (\ p.3^{n-1}k)^3 + 3.(\ p.3^{n-1}k)^2.p + 3.p.3^{n-1}k.p^2 + p^3\ (mod\ p^2.3^n)$$

$\equiv p^3.3^n k\{3^{2n-3}k^2 + 3^{n-1}k + 1\} + p^3\ (mod\ p^2.3^n)$

$\equiv p^3\ (mod\ p^2.3^n)$.

So, $x \equiv p.3^{n-1}k + p\ (mod\ p^2.3^n)$ satisfies the congruence and hence is a solution. But for $k = 3p$, the solutions formula reduces to:

$x \equiv p.3^{n-1}.3p + p\ (mod\ p^2.3^n)$.

$\equiv p^2.3^n + p\ (mod\ p^2.3^n)$

$\equiv 0 + p\ (mod\ p^2.3^n)$. This is the same solution as for $k = 0$.

Also, for $k = 3p + 1$, it is seen that

$x \equiv p^2.3^n + p.3^{n-1} + p \ (mod\ p^2.3^n)$.

$\equiv p.3^{n-1} + p \ (mod\ p^2.3^n)$. This is the same solution as for $k = 1$.

Therefore, it is seen that the solution- formula gives exactly $3p$ incongruent solutions of the congruence. So, all the solutions are given by

$$x \equiv p.3^{n-1}k + p \ (mod\ p^2.3^n); k = 0, 1, 2, \dots\dots\dots\dots\dots, (3p - 1).$$

ILLUSTRATIONS

Example-1: Consider the congruence: $x^3 \equiv 125 \ (mod\ 225)$.

It can be written as: $x^3 \equiv 5^3 (mod\ 3^2.5^2)$.

It is of the type: $x^3 \equiv p^3 (mod\ 3^n.p^2)$ with $p = 5, n = 2$.

It has $3p$ incongruent solutions given by

$$x \equiv 3^{n-1}.p^{2-1}k + p \ (mod\ 3^n.p^2); k = 0, 1, 2, \dots\dots\dots., (3p - 1).$$

$\equiv 3.5k + 5 \ (mod\ 3^2.5^2); k = 0, 1, 2, \dots\dots\dots.14.$

$\equiv 15k + 5 \ (mod\ 225)$

$\equiv 5, 20, 35, 50, 65, 80, 95, 110, 125, 140, 155, 170, 185, 200, 215 \ (mod\ 225)$.

Example-2: Consider the congruence: $x^3 \equiv 343 \ (mod\ 441)$.

It can be written as: $x^3 \equiv 7^3 (mod\ 3^2.7^2)$.

It is of the type: $x^3 \equiv p^3 (mod\ 3^n.p^2)$ with $p = 7, n = 2$.

It has $3p = 3.7 = 21$ incongruent solutions given by

$$x \equiv 3^{2-1}.7^{2-1}k + 7 \ (mod\ 3^n.7^2); k = 0, 1, 2, \dots\dots\dots., (3.7 - 1).$$

$\equiv 21k + 7 \ (mod\ 3^2.7^2); k = 0, 1, 2, 3,4,5,6, \dots\dots\dots.20.$

$\equiv 21k + 7 \ (mod\ 441)$

$\equiv 7, 28, 49, 70, 91, 112, 133, 154, \dots\dots\dots\dots\dots\dots\dots.., 427 \ (mod\ 441)$.

Example-3: Consider the congruence: $x^3 \equiv 343 \ (mod\ 3969)$.

It can be written as: $x^3 \equiv 7^3 (mod\ 3^4.7^2)$.

It is of the type: $x^3 \equiv p^3 (mod\ 3^n.p^2)$ with $p = 7, n = 4$.

It has $3p = 3.7 = 21$ incongruent solutions given by

$$x \equiv 3^{4-1}.7^{2-1}k + 7 \ (mod\ 3^n.7^2); k = 0, 1, 2, \dots\dots\dots., (3.7 - 1).$$

$\equiv 189k + 7 \ (mod\ 3^4.7^2); k = 0, 1, 2, 3,4,5,6, \dots\dots\dots.20.$

$$\equiv 891k + 7 \ (mod\ 3969)$$

$\equiv 7, 196, 385, 574, 763, 952, 1141, 1330, 1519, \dots\dots\dots\dots\dots\dots\dots.., 3787 (mod\ 3969)$.

CONCLUSIONS

Therefore it is concluded that the standard cubic congruence: $x^3 \equiv p^3 (mod\ p^2.3^n)$, p an odd prime has exactly $3p-$ incongruent solutions given by

$$x \equiv 3^{n-1}.p^{2-1}k + p \ (mod\ p^2.3^n); k = 0, 1, 2, 3, \dots\dots\dots, (3p - 1).$$

MERIT OF THE PAPER

The author's formulation made the finding of solutions simple, easy and time-saving. A large number of solutions can be obtained by the formulation. Solutions can also be obtained orally. So, formulation is the merit of the paper.

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