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A Queue Fairness Model of Automated Teller Machine (ATM) System: (A Case Study of Financial Institutions in Kaduna Nigeria)

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ABSTRACT

Until recently, consensus of opinions, thoughts and theories on the studies of ATM queuing system, and the analysis of its influence on customers' behaviour were predicated on the average time spent by a tagged customer on the queue. Very few were poised to consider the unfairness or injustice experienced by customers in the systems; in terms of whether the actual services rendered to the customers commensurate with their delay probabilities. To this end we employ the Resource Allocation Queue Fairness (RAQF) metrics to study and appraise "fairness" perceptions as indicator for service delivery quality and hence customers' satisfaction in financial institution in Kaduna metropolis. Tentatively, we evaluate both the performance measures and the unfairness characteristics of the system, with respect to Poisson arrival process and exponentially distributed service time in M/M/4 single queue system. Summarily, the results of the analyses show that, both the unfairness and the discrimination coefficients of the system in general increases with the probability of having more (k>m) customers in the system and vice versa. The high negative discrimination index as well as its corresponding high unfairness coefficient, shows that though the ATM system may be necessary to improving service delivery quality, but queuing all customers with varying job size (service requirement differences) in a single queue structure served under FCFS(job seniority) service policy is unfair and hence may provoke customers' dissatisfaction. To reverse the situation, the study recommended that (i) since all customers arriving at the system at any epoch have varying job size, then scheduling of services based on service requirement differences, and dedicating separate ATM machines to customers' with similar job-size will reduce the unfairness index and hence enhance customers' satisfaction. (ii) To reduce the ATM overutilization factor during peak periods, as well as the high delay probability associated with the system, the deployment of few ATM machines with optimal processing speed would be more cost effective than more ATM machines with relatively slow processing speed and (iii) Since the efficient utilization of the ATM machine is a function of good internet connectivity, steady electricity supply and good computer literacy level of customers, therefore, the provision of good internet connectivity, alternative steady electricity supply (solar), as well as provision of basic ATM operation guides to customers would reduce customer's sojourn time at the service point, reduce the system unfairness index and hence guarantee customer's satisfaction.

KEYWORDS: *ATM queuing system, System discrimination and unfairness index, RAQF metrics, FCFS service policy, Poisson arrival processes, service requirement differences*

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1. INTRODUCTION

Conventionally, customer or job arrivals from a finite or infinite population to a service facility often exhibit a form of queuing architecture. Waiting on such queue can sometime be an unpleasant experience which may provoke varying customers' behaviours. Therefore, the effective management of both the customers/jobs' waiting time on the queue as well as the service facility can be of immense benefit to both business organization, and the society at large. Queuing model analysis, a mathematical method of correlating customers' arrival processes on a queue if service is not immediately available with their corresponding service processes is primarily of the operations research (OR) discipline. Catalogued of queuing model applications exist in bank counters where customers await varying services; workshops where machines await repairs; warehouses where items awaits distributions; telephone exchangecentres where incoming calls await maturation before delivery, etc.[43].

In a typical resource-oriented or service delivery environment, a system serving a queue of people is a microcosm social construct where emotions and resentment may develop if delay, unfairness or injustice is practiced or is perceived to be practiced in the system, whereas courtesy and even comradeship due to prompt and equitable sharing experience may result when fairness in service is perceived. Recent studies show that the fairness issue in queuing system is much more importance to customers than the actual delays they experience [36].For instance, a company can lose patronage and or get sued if a customer loses money due to being treated unfairly on the queue. Thus, consumers' perception of both the fairness aspect and the quality of service rendered in a service delivery system may influence their reactions and continued patronage of an organization's services and products.

In reality, it is the quest to guarantee fair service delivery that informed the introduction of queues in a wide variety of fields and applications; including computer systems, telecommunication systems, and other human service delivery systems such as banks, offices, call centres, supermarkets, etc.[37,38];[51]. The correlation between queue fairness, service delivery quality and customers' satisfaction has over the recent decades appeared in scholarly publications[22];[26];[29];[47].Tentatively, it is the quest to balance fair service delivery quality with customers' satisfaction that informed the invention of the Automated Teller Machine (ATM)Technology -an innovation which has not only revolutionized the financial transactions methodologies, but also changed virtually all sphere of service delivery system[16];[48].

The ATM innovation has made the performance of activities, which were hitherto manually carried out stressfully and unproductively, much more convenient, faster, easier and more accurate. The machine has metamorphosed electronic banking system in particular to one of the most unprecedented and versatile technologies in the banking world [48]. Considering the ease with which the ATM technology allows for the transmission of data from one location to another via computers and telecommunication media, financial institutions have welcomed this innovation as a more convenient and productive approach to service delivery[41]. Considering that the major goal of the ATM innovation is to strike a balance between cost minimization in service delivery and the provision of satisfactory, reasonably quick services to customers, long queues at ATM depots across most financial institutions may incite the following unsatisfactory customers' behaviours:

- **Balking:** Some customers who are irritated by the long queue may not join at their precise position but attempt to jump to close proximity to the service point by passing others ahead of them.
- **Renege:** Some customers who join the queue for some time and then gets irritated and leaves the queue after some time.
- **Collusion:** Some of the costumers may conspire to present only one customer on the queue instead of all queuing. However, when it is their turn to be served, all the costumers in the collusion demand for service.
- **Jockeying:** When more than one queue dispense similar services, some costumers may skillfully maneuver their way from one queue to another in order to get in close proximity to service point before their normal time.

Though the implications of customers' waiting time on the ATM queues have over the recent decades attracted many research studies [1];[3];[9];[42];[44], however, the unfairness aspect inherent in queuing systems has hardly been studied or quantified. The conjecture about the queue fairness and customers' satisfaction has not been empirically tested in service delivery context (e.g. financial institutions). Thus, considering that customers entrusting their financial resources to banks deserve fair treatment commensurable with their waiting time probability, queue fairness plays a critical role in determining customers' satisfaction in the financial services industry[30];[32].

It is on this premise that this study seeks to employa quantitative queue fairness metrics proposed in the literature by Raz et al[37,38], to study and quantify "queue fairness" perceptions, as indicator for service delivery quality and hence customers' satisfaction in financial institution in Kaduna state, Nigeria. A survey of most banks' ATM queuing system present a configuration at least four (4) parallel ATM machines servicing streams of randomly arrived customers under a single First-Come-First-Serve (FCFS) queuing policy. This queuing architecture has attracted a lot of criticisms over the last decades; hence, the appraisal of their performance measures is an inevitable undertaking. Specifically, the study argues that queue fairness perceptions, which led to higher customers' trust and in turn affecting consumers' satisfaction, may have been compromised or slaughtered on the altar of pure service delivery quality in these systems. Hence, emphasizing the significance of queue fairness appraisal as a complementary component to service delivery quality determinant in the institution.

1.1. Statement of the Problem

A careful observation and examination of most banks' ATM centres within the Kaduna metropolis, reveals perennial long queues of customers at most peak hours of the days, while the machines often remain idle at few non-peak hours. As a result of such scenario, customers waste lots of productive times on the queue (opportunity cost of time spent in queuing).Oftentimes, customers grew annoyance, dissatisfaction and balk or renege from the system. These long queues and consequences thereof could be managed to enhance quality service delivery if proper appraisal and analysis of the systems are accorded the priority they deserved. Predicated on this background, the present study seeks to appraise the performance measures as well as the queue fairness coefficient of some financial institutions in Kaduna metropolis; with a view to determining a possible queuing architecture or otherwise, that would minimize the perennial long queues, as well as promoting customers' satisfaction and service delivery quality.

To embark on this onerous academic exercise, we survey, observed and gather the relevant data and information on system weekly operations from the respective respondents (bank Management). Subject to peculiarly emotion and prejudice from the respondents the accuracy of the research data could be disputed. However, the extent to which these attributes could impact on the efficacy of the research results and findings have been significantly reduced through collaborative efforts of professional authors in the field of operations research (OR) and queue model analysis. The use of fairly large number of inputs from twenty (20) financial institutions' weekly operational records and cross referencing same with at least 4 weeks (28 days) of personal observation of the system also add credence to our research data and results. Furthermore, the data collected from the respondents were also cross-referenced with records from similar financial institutions outside the metropolis. While the institutions' records of service distribution for five (5) consecutive business years were equally used to corroborate and fill in any observed lapses.

2. RELATED ACADEMIC LITERATURE

The improvement of performance and service quality in service industries when jobs/customers' arrival times and their corresponding service times are randomly distributed is a complex decision environment, especially when functions are performed manually by human employees[7];[42]. This scenario can best be experienced in modern financial institutions e.g. banks; where there is a daily influx of customers for diverse financial transactions. Therefore, the deployment of Automatic Teller Machine (ATM) technology to complement human effort is among the most important service facilities in the contemporary banking industry[7];[45]. Though scholars are unanimous on the indispensability of the ATM technology in actualizing the Cashless Policy of many nations, however, this is certainly not without challenges[7]. Thus, the ATM innovation which was aimed at enhancing service delivery quality by making job performance much more convenient, faster, easier and more accurate has in itself resulted in large service demands which directly translate to queues when these demands cannot be quickly satisfied.

The psychological traumas which are characterized by congestion, long queue lengths, long waiting times as well as over utilizations of ATMs across commercial banks in Nigeria calls for concern. These challenges which are traceable to the recent upsurge in the customer base of most banks without equivalent increase in service capacity[46],have become more evident during weekend periods and month endings (salary payment periods) where the demand for cash from banks is high; hence scholars' probe into the performance indices of ATMs queuing architecture.

Applying the M/G/1 queue model to study the ATM queuing systems across selected banks in Nasarawa state, judging from the ATM service rates, authors has criticized the cost ineffective solution of multiplying the number of ATM machines instead of installing fast ATMs or upgrade the speed of existing ones[7]. Other researchers also assumed stochastic birth-death Markov processes with Poisson arrival and exponential service time distributions of the respective single-server and multi-server (M/M/1 and M/M/c) architectures, to address the problem of long waiting time of customers and server over utilization. But overlooking the cost implications of purchase, installation and maintenance of the additional machines, these authors were unanimous in the increment of the number of ATM machines in banks[18];[31];[49]..

In an attempt to address the expected waiting time of a customer in the queuing system, some authors have focused on the banks queuing system. That is the correlation between the different queuing algorithms that are used in banks to serve the customers, and their average waiting time characteristics. The aim of these works was to build an automatic queuing system for organizing customers, analyse the queue status and making decision on which customer to serve at any point in time[33]. The new queuing model was expected to switch between different scheduling algorithms according to the test results and the factor of the average waiting time. The main innovation of this work includes modelling the average waiting time taken into processing, in addition to the process of switching to the scheduling algorithm that gives the best average waiting time. Under the job scheduling paradigm, a comparative analysis of FCFS and LCFS queuing policy in bank queues, shows that the particular queuing discipline chosen for the service may greatly affect customers' waiting time; as no one would want to arrive early in a LCFS discipline. However, the LCFS discipline doesn't generally affect the important outcome of the queue itself, as arrivals are constantly receiving service[11]. Finally, researchers also emphasizes the relevance of queuing model analysis for effective service delivery of the Nigerian banking sector and strongly recommends same for efficiency and quality of service delivery to managements of GT Bank and Eco bank[17].

As observed above, a host of queue performance measurements are predicated on the delay distribution perspective, i.e. the average time spent by a tagged customer on ATM queue system. However, very few if any are poised to consider queue fairness in terms of whether the actual services rendered to the customers commensurate are with their delay probabilities. Recent studies show that the fairness issue in a queuing system is of much importance to customers than the actual delays they experience[28]. Specifically, the present study argues that queue fairness perceptions which lead to higher customers' trust and in turn affect consumers' satisfaction is a complementary component to service quality determinant in any service industry.

2.1. The Concept of Queue Fairness

The issue of fairness or social justice has always been and is still a cardinal issue in all cultures and traditions; as it is the cement holding the society's fabric together. As expected, a large volume of literature on the subject matter of fairness doctrine abound, however, a more prominent postulation on fairness issue is the "Theory of Social Justice", whose general conception stipulates that: *"All social primary goods such as liberty and opportunity, income and wealth, and the bases for self-respect, should be distributed equally unless an unequal distribution of any or all of these goods is to the advantage of the least favoured"*[39].Corroborating this assertion, Aristotle[8]once motioned that, *"like cases should be treated alike and different cases should be treated differently in proportion to their differences"*. Others scholars also observed that *"a fair decision is one that makes for proper balance of conflicting interest"*[10].The proper balance or fair share of any resources, according these schools of thought is determined by consulting the majority, employing logic and or referring to the divine. We therefore see that throughout history, one prominent concept in human tradition is that of fair resource allocation, or the idea that fairness should be the watch-word when a system resource is divided between contending consumers.

2.2. Queue Fairness as Indicator of Customers' Satisfaction

What are the resources that worth a fair division in the case of a queuing system, how and why should it be divided fairly among contending customers? As embodied by the ideal Processor Sharing (PS) policy studied as early as Kleinrock[23] and Coffman et al[14] works, the root idea is that *"at every moment of time, the servers' service rate should be divided equally amongst the jobs/customers present in the system....and a violation of this policy connotes unfairness to the customer"*. This exactly reflects the idea behind

RAQF metrics, whose basic principle stipulates that*: "at any epoch of time, all jobs/customers present in the queuing system deserve an equal share of the system's service rate… and deviations from it create discriminations (positive or negative)"*[37,38]. And accounting for these discriminations with summary statistics yields a measure of unfairness for the system.

The fairness factor associated with waiting on queues has been recognized in many works and applications. Larson[25] in his discussion paper on the dis-utility of waiting recognizes the central role played by "Social Justice", which is another name for fairness, and its perception by customers. Rothkopf and Rech[40] also addressed this in their paper discussing perceptions in queues; where an impressive list of quantifiable considerations shows that combining queues may not be economically advantageous, contrary to the common belief, though such considerations may not have sufficient weight to overcome the unfairness perceived by customers served in separate queues structures.

Aspects of fairness in queues were also qualitatively discussed earlier by quite a number of authors. Palm[35] judges the annoyance caused by congestion, Nikolić and Cvejić[32], discusses the queue as a social system, and Whitt[50] addresses overtaking in queues. Scientific evidence of the importance of fairness of queues was also provided by Rafaeli et al[36], which employs experimental psychological approach to study the reaction of humans waiting on queues under various service scheduling policies. The impactful study revealed that for humans waiting on queues, the issue of fairness is highly important, sometimes even more important than the duration of the wait. For the case of common queue versus separate ones at each server, the authors observed that the common queue is fair than separate queue structure. Probably, for this reason we find separate queues mostly in systems where a common queue is physically not practicable, such as traffic toll booths and supermarkets. In addition, the issue of fairness was discussed in the context of practical computer applications and web servers by Harchol-Balter et al[21], where a queuing policy was observed to have reduced response times but at the expense of unfairness to large jobs.

2.3. The Determinants of Queue Fairness

Researchers in the recent decades have argued that individual judgments of a firm's service-related outcomes should increase as the firm achieves higher levels of fairness in its service delivery system[6]. Prior research has shown that suppliers' fairness arbitrates the relationship between supplier performance and reseller satisfaction[52]. Both fairness and system success factors influence users' satisfaction and continued intention on web-based learning systems[13]. Fair treatment received from the exchanging party also influences customers' satisfaction and loyalty intentions of online shoppers[15]. Customers' perceptions of the level of queue fairness have a positive effect on their perceptions of service delivery quality[12]. In other words, queue fairness is directly related to customers' satisfaction and there is a positive correlation between perceived queue fairness, customers' satisfaction and service delivery quality.

2.3.1. Customers Satisfaction:

Customers' satisfaction has long been recognized as a central concept and a critical goal of all business activities. A preponderance of evidence supports the significant relationship between a company's financial performance and the satisfaction of its customers [29]. Customers' satisfaction increases favourable behavioural intention regarding the service providing units. For example, research shows that satisfied customers stick with some specific firms, and are more willing to provide feedback and make recommendations through word of mouth[19];[35]. Customers' satisfaction is often defined as a judgment based on one or series of consumers' service interactions. Other researchers have argued that satisfaction is based on comparison with pre-consumption standards [33].

However, most research into the parameters that influence the levels of consumers' satisfaction and retention in financial institutions often focuses on service delivery quality[5]. Though service quality literature also argued that perceived service quality performance is the most powerful predictor of customers' satisfaction[24].On the contrary, scholars like Oliver &Swan[33]have suggested that perceived justice (fairness) should be considered in appraising consumers' satisfaction. Equity or fairness has been suggested as a standard for measuring employees' satisfaction [12]. Therefore, this study argues that simply escalating service delivery quality may not be the best approach to strengthen consumers' satisfaction. Instead, fairness and the associated customers' attitude might be a major influence on consumers' satisfaction; hence, the quantification of queue fairness should also be a priority performance measure in service industries.

2.3.2. Service Delivery Quality

Service delivery quality and customers' satisfaction are key factors of business competition for manufacturers and service providers alike[27]. With the increasingly intense competition for customers in today's service-oriented institutions, these factors have become managements' high priorities[34]. Researchers generally agree that service delivery quality leads to higher levels of customers' satisfaction. Hence financial executives and banking strategists have become more focused on service delivery quality to increase customers' satisfaction as well as business success in the financial institutions[4]. Until recently, no study has challenged this dominant conceptualization of service delivery quality as the major determinant of customers' satisfaction in service delivery system. Research scholars have argued that managers cannot simply create services, provide them at high quality levels and hope for the best, instead service delivery quality evaluations should result from a comparison of service delivery against the norms of fairness and the treatment of similar customers contending for the services[12], hence, the queue fairness consideration. The queue fairness consideration, which is a derivative of this claim, is developed from the insight that consumers' reactions to services are, at least, and in part based on equity theory[2]. This means that consumers are also interested in equitable quantity rather than only favourable quality treatment. For example, the reactions of customers at ATM queuing system may not be unconnected with their perceptions of whether they have been treated fairly by the system. Perhaps the major reason for using an ordered queue in service delivery system at all is to provide fair queuing to the customers. In this sense one can view a queue as a "fairness management facility".

2.4. Measuring Queue Fairness

Until recently, consensus of opinions, thoughts and theories on the studies of queuing system, its quantifications as well as the analysis of queue fairness and its influence on customers' behaviour were predicated on the delay distribution perspective - that is, the average time spent by a tagged customer in the queuing system[20];[23];[25]. Very few if any were poised to consider queue fairness or injustice experienced by customers in queuing systems in terms of whether the actual services rendered to the customers are commensurate with their delay probabilities. Predicated on this background, Raz et al[37,38] proposed RAQF metrics, whose underlying principle involves the sharing of the system resources equally among contending customers at any epoch of time. RAQF model which is based on the application of the basic principle of social justice to the system, demands that *"equally needy members of a group should share equally the resources available to the group"*. Thus, RAQF basic philosophy demands that *"at every epoch at which there are N(t) jobs/customers present in the system, they all are entitled to an equal share of the server's time (or product)"*, and deviations from this principle result in discrimination (positive or negative). A unique property of RAQF metrics is the tracking of jobs/customers inter-relationship and the resulting unfairness throughout the queue progress process, thus, allowing for understanding and evaluation of fairness at both the individual customer's level and the unfairness of specific scenario, as well as the overall unfairness of the system or policy.

2.4.1. Fundamental Principle of Queue Fairness Measurement:

Major proponents of queue fairness measures[10]; [36]; [37,38]are unanimous on two fundamental quantities that determine queuing process and job scheduling policies. These are the arrival epochs and service times of the job/customer. Given the importance of these quantities to queuing system, they also serve as the fundamental variables for determining queuing fairness. For convenience of presentation, these quantities are also termed job seniority (arrival time), and service requirement (job size) as established by the following conceptual frame work.

2.4.2. Conceptual Framework of Queue Fairness Measurement:

Consider a general queuing system consisting of a single server, customers $C_i, i = 1,2, \dots$ arrived the system at arbitrary arrival epochs, $a_i, i = 1,2, \dots$, respectively; where $a_i \leq a_{i+1}$. Customer $C_i$requests some service at the server, the amount of which is denoted by s_i, measured in units of time and the server grants service to the customers according to some scheduling policy π. Once $C_i$receives its full amount of services_i, (which may not be given continuously or at full rate) it leaves the system at an epoch d_i, called its departure epoch. The duration $C_i$stays in the system denoted by $t_i = d_i - a_i$ is called the system time and the duration $C_i$waits and does not get service denoted by $\omega_i = t_i - s_i$, is called the waiting time of C_i(except for processor sharing disciplines where this conventional definition of waiting time may not be applicable). The variables a_i, s_i, d_i, t_iandω_i represent the actual values attributed to $C_i$in a specific sample path of the system.

Therefore, the *seniority* of customer $C_i$at epoch *t* is given by$t - a_i$, while the *service requirement* of customer $C_i$at epoch *t*is s_i. It is natural to expect that a "fair" service policy will give preferential service to highly *senior* customers and low *service-requirement* customer. This can be stated formally in the following fundamental principles:

A. Service-Requirement Preference (SRP) Principle:

If all customers in the system have the same arrival time, then for customer $C_i$and C_j,arriving at the same time and residing concurrently in the system, if $s_i < s_j$, then it will be more fair to complete service of $C_i$ahead of $C_j$than vice versa.

B. Jobs Seniority Preference (JSP) Principle:

If all customers in the system have the same service times, then for customer $C_i$and C_j,residing concurrently in the system, if $a_i < a_j$, then it will be more fair to complete service of $C_i$ahead of $C_j$than vice versa.

The JSP principle is rooted in the common belief that customers arriving at the system earlier "deserve" to leave earlier. While the SRP principle is rooted in the belief that it is "less fair" to have short jobs wait for long ones, it should be noted that when $a_i < a_j$ and $s_i > s_j$or $a_i = a_j$ and$s_i = s_j$, the two principles conflict each other.And thus, the relative fairness of the possible scheduling of customer $C_i$and $C_j$is likely to depend on the relative values of the parameters. One may view these two preference principles as two axioms expressing one's basic belief in queue fairness. As such, one may expect that a fairness measure will follow these principles. A fairness measure is said to follow a preference principle if it associates higher fairness values with scheduling policies that are more fair. A formal definition may be stated thus:

Axiom 2.0: JSP Principle: Consider customers $C_i$andC_j, requiring equal service times and obeying $a_i < a_j$. Let πbe a scheduling policy where the service of $C_i$is completed before that of $C_j$and $\pi' = \pi$except for exchanging the service schedule of $C_i$and C_j.A fairness measure is said to adhere to the seniority preference principle if the fairness value it associates with π is higher than that which it associates withπ'.

Axiom 2.1: RSP Principle: Consider customer $C_i$and C_j, arriving the same time at the system and obeying: $s_i < s_j$. Let πbe a scheduling policy where the service of $C_i$is completed before that of $C_j$and $\pi' = \pi$, except for exchanging the service schedule of C_i and C_j. A fairness measure is said to adhere to the service-requirement preference principle if the fairness value it associates with π is higher than that which it associates withπ'.

3. THE AUTOMATED TELLER MACHINE AS QUEUE FAIRNESS INNOVATION

Fairness in the allocation or distribution of resources (time, products or service) in service industries is a fundamental issue in human society, perhaps the major reason for employing ATM technology in the queuing system of most financial institutions[42]. Hence one can view the ATM queue as a "fairness management facility". Multi-server single-queue FCFS queuing policy is one queuing architecture or strategy employed by most financial institutions in Kaduna metropolis to provide fair service delivery to its teaming customers. Service requirement(job size)as one of the fundamental quantities of queuing process and scheduling policy as well as its influence on the system, particularly, the fairness aspect, which is highly important to customers has always been sacrificed on the altar of resource optimization inmost bank ATM queuing systems**.** Therefore, the main focus of this chapter is to apply RAQF metrics [37,38], which accounts for both job seniority (arrival time) and service time differences, to evaluate the unfairness factors of these systems. RAQF metrics which focus on examining how the ATM resources (service rates) are fairly allocated to the customers, demands that:*"at every epoch, all customers present in the system deserve an equal share of the server's resources (attention) and that deviation from this standard creates discrimination (positive or negative)*", and accounting for these discriminations and their summary statistics yield a measure of unfairness. To this end we review and appraised organizations' ATM Centres with RAQF system metrics of multi-server single-queuing systems under Poisson arrival and exponential service time distributions(M/M/m).

3.1. System Model of RAQF Metrics

Consider a work conserving non-idling system, M/M/m, with k parallel servers, $k = 1,2,\ldots,m$ serving a stream of customersC_i; $i = 1,2,\ldots.$, whose arrivals follow a Poisson process with rate λ, and their required service times are identically independent random variable $(i.i.d)$ exponentially distributed with mean μ. By Sztrik[47] the system traffic intensity:

$$\rho = \frac{\lambda}{m\mu}; \leq m \; (3.0.0)$$

Let a_i and d_i denote the arrival and departure epochs of C_i respectively. Let s_i denote the service requirement (measured in time units) of C_i, and let $\omega(t)$ denote the total service rate granted by the server at any epoch (which usually is an integer equalling either the number of working servers or the total servers' time or its resources at that epoch), and $N(t)$ denotes the number of customers in the system at that epoch. Then by RAQF measure, the fair share called the *momentary warranted* service rate of $C_i$is given by

$$R_i(t) = \frac{\omega(t)}{N(t)} \; (3.0.1)$$

Let $\sigma_i(t)$ be the momentary granted service rate of $C_i$at epoch t, then the *momentary discrimination* rate of C_i at the epoch when C_i is in service denoted by $\delta_l(t)$ is given by:

$$\delta_i(t) = \sigma_i(t) - R_i(t) = \sigma_i(t) - \frac{\omega(t)}{N(t)} (3.0.2)$$

Equation (3.0.2) can be viewed as the rate at which discrimination accumulates for C_i at epoch t.Let$\delta_i(t) \stackrel{\text{def}}{=} 0$, if $C_i$is not in the system at epoch t,but we are only interested in $\delta_i(t)$when C_i is in the system. Thus, the total discrimination of C_i denoted D_i is:

$$D_i(t) = \int_{a_i}^{d_i} \delta_i(t)dt \; (3.0.3)$$

A positive or negative value of D_i means that a customer received better or worse treatment than it fairly deserves, and therefore it is positively or negatively discriminated.

3.1.1. Alternative RAQF Metrics Formulation:

The definition of the momentary warranted service and discrimination, given in equations 3.0.1 and 3.1.2 above is based on the concept that a customer deserves an equal share of the *resources granted* $\omega(t)$by the system at that epoch, and any deviation from it creates discrimination among the customers residing in the system. However, if some of the resources are not granted at that epoch, e.g., due to system idling, or due to the use of only part of the servers, it may be considered as being *inefficient*, but not as discrimination and unfairness. One could consider an alternative concept by which at any epocha customer deserves an equal share of *all the available system resources.* Under such notation, the warranted service will be defined as

$$R_i(t) = \frac{m}{N(t)} (3.0.4a)$$

And the momentary discrimination replaced by,

$$\delta_i(t) = \sigma_i(t) - \frac{m}{N(t)} (3.0.4b)$$

The difference between the two alternatives is conceptual and relates to situations where the system does not grant all of its resources. One such case is a multi-server system at epochs where the number of customers is smaller than the number of servers ($N(t) < m$. Another case is a system which allows server idling - when there are customers in the system.

This issue and the tradeoff between the alternative formulations are more pronounced in multi-server multi-queue systems. However for this work, we choose to focus on the concept of fair division of the *granted* resources (equation 3.0.2), as this is more appealing since the cases where the system does not grant all resources are limited to situations that result from system operations constraints (system cannot serve a single customer by many servers), and thus may possibly be interpreted by customers as non-discriminatory. For work conserving systems (systems in which the total service given to a customer over time equals its service requirement), i.e.,

$$\int_{a_i}^{\infty} \sigma_i(t) = s_i(t)$$

We have accumulative discrimination of C_i

$$D_i(t) = s_i(t) - \int_{a_i}^{d_i} \left(\frac{\omega(t)}{N(t)}\right) dt = \int_{a_i}^{\infty} \sigma_i(t) - \int_{a_i}^{d_i} \left(\frac{\omega(t)}{N(t)}\right) dt \; (3.0.5)$$

A positive or negative value of $D_i(t)$ mean that a customer received better or worse treatment than it fairly deserves and therefore it is positively or negatively discriminated. An important property of this measure is that, for every non-idling service conserving system and for every *t*: $\sum_i D_i(t) = 0$, (i.e.*every positive discrimination is balanced with equivalent negative discrimination*). As observed in Raz et al[37,38] for a single server non-idling work conserving system, the expected value of discrimination always obeys$E[D] = 0$. Thus, the unfairness of the system is defined as the second moment of the discrimination, $E[D^2]$. The same property alsoholds in multiple server non idling systems.

3.2. The System Unfairness Coefficient

Let $E[D^2|k]$; $k = 0,1,2, \dots$ denote the expected value of the square of discrimination, given that customer C_i encounters k customers in the system on arrival (including the ones being served). Let P_k be the steady state probability that there are $k$customers in the system. By PASTA property, this is also the probability that k customers are seen by an arbitrary arrival, hence, the second moment of D (the unfairness) is given by:

$$E[D^2] = \sum_{k=0}^{\infty} E[D^2|k]P_k \quad (3.0.6)$$

Where,$P_k$is the steady state probability of a single queue M/M/m system and is given by:

$$P_k = \begin{cases} P_0 \dfrac{[m\rho]^k}{k!}; k \le m \\ \qquad\qquad\qquad \text{and } P_0 \\ P_0 \dfrac{\rho^k m^m}{m!}; k \ge m \end{cases} = \left[\frac{[m\rho]^m}{m!\,[1-\rho]} + \sum_{k=0}^{m-1} \frac{[m\rho]^k}{k!} \right]^{-1} \quad (3.0.7)$$

Where $m$denotes the number of parallel servers, k denotes the number of customers in the system and P_0 denotes the probability of an empty system. By Sztrik[47]the overall system unfairness, denoted by$Var[D]$- the variance of the system discrimination coefficient is given by:

$$Var[D] = E[D_i^2] - [E[D_i]]^2$$
$$= \sum_{i=1}^{k} E[D_i^2] - \left[\sum_{i=1}^{k} E[D_i]\right]^2 \quad (3.0.8)$$

The validity and reliability of equation (3.0.8) can be determined in confidence interval:$CI = D_i \pm t\alpha/_2\sqrt{Var[D]}$; where $\alpha = 0.05\ (5\%)$ level of significance.

4. DATA PRESENTATION AND ANALYSIS

Based on observations and records of past service distribution(table 1.0) obtained from twenty (20) financial institutions within the Kaduna metropolis, we assumed that the case ATM system represented by figure 1.0 below as a queuing system, is synonymous to a $M/M/m$ single queuing system. It specifies a multi-server queuing mathematical model, where the two *Ms* denotes the respective Poisson arrival and exponential service time distributions respectively, and $m = 4$represents the number of parallel ATM machines in the system. Customers arrive randomly at the ATM system to seek different forms of services (e.g. cash withdrawals, transfer, pay bills, etc.). Particularly, customers arriving for cash withdrawal seek varying size of service time from the system. The ATM system operates in such a manner that for each arrival, if all the machines are busy then the customer enters the single queue; else the arriving customer immediately enters service. Ironically, service scheduling is only base on customer's arrival time (job seniority), with no consideration given to job size (service requirement), thus, treating all customer as a single class as depicted by figure 1.0 below.

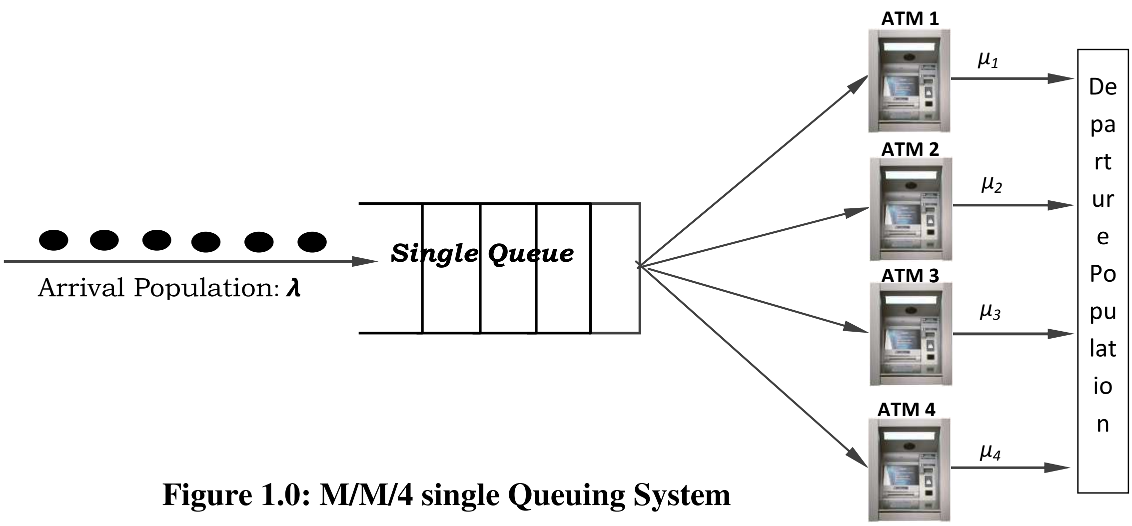
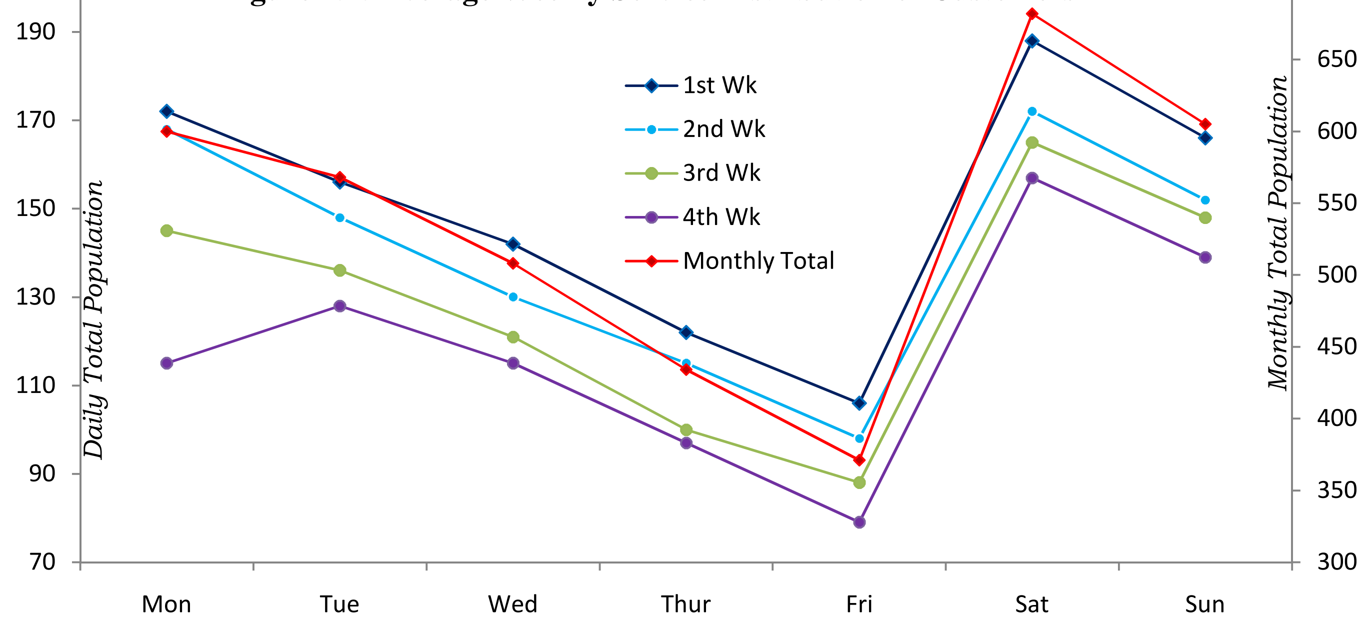


Figure 1.0: M/M/4 single Queuing System

Table 1.0: Average Weekly Arrival of Customers' Population

Days / Week	Working Days					Weekend		Weekly
	Mon	Tue	Wed	Thur	Fri	Sat	Sun	Total
1st Wk	172	156	142	122	106	188	166	1052
2nd Wk	168	148	130	115	98	172	152	983
3rd Wk	145	136	121	100	88	165	148	903
4th Wk	115	128	115	97	79	157	139	830
Monthly Total	600	568	508	434	371	682	605	3768

Figure 1.1: Average Weekly Service Distribution of Customers



The figure 1.1 above shows that, the mean number of customers on weekends (Saturday and Sunday) is double that on working days during the month. The busiest days of the week for the ATM servers are Saturday, Sunday and Monday, while the first week of the month also exhibits the highest customers' traffic on the system. This may not be unconnected with the usual upsurge of customers at the ATM during weekend periods where demand for cash for family expenses is high, and also first week of the month due to the usual late payment of civil servants' salaries in Nigeria. The figure 1.1 also shows that, after Monday, the number of customers started to decline slowly as the week progresses, as Fridays exhibit the least customers' traffic due to the usual Friday Jumat prayer services of the Muslim customers.

4.1.1. Model Assumptions:

The model above is predicated on the following assumptions in accordance with the queuing theory:

- Poisson arrival rate of customers per unit of time,
- Exponential service times of customer per unit of time,
- Identical service facilities (since same kind of transactions are performed on all ATMs),
- A queue with unlimited waiting space that feeds into identical servers,
- Queue discipline is FCFS basis on single queue structure,
- There is no limit to the number on the queues (infinite),
- The average arrival rate is greater than average service rate.

4.1.2. Determining Performance Measures of the Model:

This study is most interested in two performance measures (i) the operating characteristics of the system, and (ii) the unfairness characteristics of the system. The key operating characteristics of interest is the customer waiting time which consists of the time a customer spends in the queue and the total time a customer spends in the system. Since we are dealing with human beings, then specifically a customer waiting time in queue is key to this performance measure. It is waiting-in-queue that is dissatisfactory to customers and greatly affects their service experience. Other parameters of interest to the ATM activity include the percentage of time they may be busy or idle. The following performance measures of the system would be computed:

- λ: the mean customer arrival rate per minute,
- μ: the mean customer service rate per minute,
- ρ: the system traffic intensity - the probability that the ATMs are busy at random time *(t)* within the interval,
- L_s: the average number of customers in the system per time period,
- L_q:the average number of customers in the queue per time period,
- W_s: the average customer's waiting time on the ATM terminal (including the waiting time in queue),
- W_q: the average customer's waiting time in the queue,
- P_0: Probability of empty system,
- $\omega(t)$: total warranted service rate of customers per day,
- $R(t)$: the momentary warranted rate of customers per server per day,
- $\sigma(t)$: the momentary granted rate of customers per server per day,
- $\delta(t)$: the momentary discrimination of a customer per server per day,
- $D(t)$: the accumulative discrimination of customers per server per week,
- $E\left[\widetilde{D}_{(k)}\right]$: the system discrimination given that $k \leq m$ customers are in the queue,
- $Var[D]$:the system unfairness coefficient given that $k \leq 4$ customers are in the queue.

4.2. Operating Characteristics of the System

Given that the four ATM servers run for a maximum period of 14 hours per day (8am to 10pm);using data from table 1.0 above the average customers' arrival rate of the system:

$$\lambda = \frac{3768\,customers}{14hrs\ (28days)} = \frac{3768}{392} = 9.612\ \approx 10\ customers/hr$$

Thus, at every hour an average of 10 customers must arrived the system for service. From observation and discussion with the relevant security guards that an ATM knowledgeable customer spends an average of 1.5 minutes on the ATM machine before departure, thus, giving an average service rate:$\mu = 10$ customers per hour.

By equation (3.0.0), the traffic intensity of the M/M/4 system:$\rho = 0.2403$. This gives an average of 24.03% customers per busy ATM per hour, while the system utilization rate$(1 - \rho = 0.7597)$, implies that about 75.97% of the customers are on queue per hour. If the additional time required by the customer to complete his/her services is exponentially distribution with mean$(\rho^{-1} = 4.16)$, then a customer will spent an average time of 4.16 minutes from entering the system to departure. The system throughput or the mean number of requests serviced per a time unit$(\gamma = m\rho\mu = 4(0.2403)(10) = 9.612\ \approx 10$ customers per hour. Thusan average of 10 customers' services is granted per hour. By equation$(3.0.7)$, the probability that the system is empty, i.e. there is no customer in the system,

$$P_0 = \left[\frac{[4(0.2403)]^4}{4!\,[0.7597]} + \sum_{k=0}^{3} \frac{[(4)0.2403]^k}{k!}\right]^{-1} = 0.382$$

This implies that the system is always idle at only 38.2% of the time, while getting busy with customers at$1 - P_0 = 0.618(61.8\%)$ of the time. We also found out that an average queue length on busy days is about8customers: $\left(L_q = 8\right)$. Theoretically, from Little's theorem[47]the average waiting time of a customer in the queue:

$$W_q = \frac{L_q}{\lambda} = \frac{8}{10} = 0.8hrs = 48mins$$

The average waiting time of a customer in the system:

$$W_s = W_q + \frac{1}{\mu} = 0.8 + 0.1 = 0.9hrs = 54mins$$

Therefore, the average number of customers in the system

$L_s = \lambda W_s = 10\ (0.9) = 9$ customers,

By equation (3.0.7), the probability that an arriving customer will meet $k \leq m$ customers in the system:

$$P_k = P_0 \frac{[m\rho]^k}{k!} = \frac{0.382[0.9612]^k}{k!}; k \leq m\ (3.0.9)$$

We assume that an arriving customer will join the queue if he/she meets $k \leq m$ customers in the queue. Suppose the system is not empty, then an arriving customer must join the queue if it meets$k \leq m$customers in the system, otherwise he/she balk or renege. Since the ATM's capacity is 4 customers at any epoch, we can calculate the probability that an arriving customer meet $k \leq 4$customers in the system

$$Prob\ (a\ customers\ will\ enter\ system) = Prob\ (At\ least\ one\ customer\ in\ queue)$$
$$= Prob\ (At\ most\ 4\ customers\ in\ system)$$

$$P_{k\leq 4} = \sum_{k=1}^{4} P_k = \sum_{k=1}^{4} \frac{0.382[0.9612]^k}{k!} = 0.6138 = 61.38\%$$

Thus, on less busy days, an arriving customer will enter the queue at61.38% of the time, or balk or renege at 38.62% of the time. Similarly, by equation(3.0.7), the probability that an arriving customer will meet $k \geq 4$ customers in the system is given by:

$$P_k = P_0 \frac{\rho^k m^m}{m!} = 4.075[0.2403]^k;\ k \geq m\ (3.1.0)$$

We also assume that an impatient customer will balk or renege if he/she meets$k \geq 4$ customers in the system. Suppose the maximum queue length that a patient customer can tolerate is 8customers. Since the capacity of the ATM is 4customers at any epoch, we can calculate the probability that an arriving customer will meet at most 8customers in the system. Therefore,

$$Prob\ (a\ customer\ will\ balk\ or\ renege) = Prob\ (At\ least\ 4\ customer\ in\ queue)$$
$$= Prob\ (At\ most\ 8\ customers\ in\ system)$$

$$P_{k\geq 4} = \sum_{k=4}^{8} P_k = \sum_{k=4}^{8} 4.075[0.2403]^k = 0.018 = 1.8\%$$

Thus, on busy days, an arriving customer will balk or renege at1.8% of the times, orenters the system at 98.2% of the times. Finally, by Erlang loss formula, the probability that an arriving customer must wait or balk or renege (delay probability)[47],

$$D = C(m,\rho) = \frac{\rho(m-1-\rho)\cdot C(m-1,\rho)}{(m-1)(m-\rho) - \rho C(m-1,\rho)} \quad (3.1.1)$$
$$= \frac{(0.2403)(2.7597)\cdot C(3,0.2403)}{3(3.7597) - (0.2403)C(3,0.2403)} = \frac{0.66315591(1.4761)}{11.2791 - 0.35470683} = 0.0896$$

Thus, on less busy days, an arriving customer must wait for 8.96% of the time in the queue before entering into service or balk or renege. While on the busy days an arriving customer must wait for 91.04% of the times on queue before entering into service.

4.3. Unfairness Characteristics of the System

Given that the four (4) parallel ATM servers run for 14 hours per day for 7 days of the week, and served a single queuing of customers on FCFS scheduling (job seniority preference) policy. Therefore, the total warranted service rate of the system per day is$\omega(t) = 4(14) = 56$hours per day or 560 customers per day. By equation (3.0.2), the momentary warranted service of each ATM machine $R(t) = \frac{560\ \text{customer}}{4\ servers} = 140$ customers per server per day. But from table-1, the momentary granted rates of each ATM machine: $\sigma(t) = \frac{3768\ \text{customers}}{28\ days(4\ servers)} = 33.6 \approx 34$ customers per server per day. By equation (3.0.2), the momentary discrimination (deficient service) the system: $\delta(t) = 34 - 140 = -106 < 0$ customers per server per day. This implies that 106 customers are dissatisfied with the bank ATM service per day.And by equation (3.0.3), the accumulative discrimination (under service) of the system over its 7-day operation: $D(t) = -742 < 0$, customers per server per day. This implies that 742 customers are dissatisfied with the bank ATM service per week. Therefore, the accumulated discrimination of the 4 ATM machines over the 28 working days:

$$D = \sum_{i=1}^{28} D_i = -742(28)(4) = -83{,}104\ customers$$

This means a highly discriminative queuing system; where 83,104 customers are dissatisfied per monthly.

4.3.1. System Discrimination Coefficient:

Let P_k denote the steady state probability that there are $k$customers in the system at any epoch. Therefore, from the "discrimination version" of Little's Theorem[47] the weekly discrimination index of the system, given that an arriving customer meets $k \leq 4$ customers in the system: $E\left[\widetilde{D}_{(k)}\right] = \boldsymbol{\lambda} E\left[D_{(k)}\right]$is given by table-2.0 below:

Table 2.0: System Discrimination With Respect to $P_{k\leq 4}$ Customers in System

No. of Customers:	1 Customer	2 Customers	3 Customers	4 Customer	$P_{k\leq 4}$
Prob of k-Customer:	0.3672	0.1765	0.0565	0.0136	0.6138
$E\left[\widetilde{D}_{(k)}\right] = \lambda[DP_k]$:	-2,724.62	-1,309.63	-419.23	-100.91	-4,554.39

The table 2.0 above shows that, given 61.38% probability that an arriving customer will meet $k \leq 4$ customers on less busy days, the system discriminative index: $E\left[\widetilde{D}_{(k)}\right] = -4{,}554.39 < 0 \approx -4{,}554$, thus, a high discriminative system. Similarly, the weekly discrimination index of the system, given that an arriving customer meets $k \geq 4$ customers in the system is given by table-2.1 below:

Table 2.1: System Discrimination With Respect to $P_{k\geq 4}$ Customers in System

No. of Customer:	4 Custs	5 Custs	6 Custs	7 Custs	8 Custs	$P_{k\geq m}$
Prob of k-Custs:	0.0136	0.0033	0.0008	0.0002	0.00005	0.018
$E\left[\widetilde{D}_{(k)}\right] = \lambda[DP_k]$:	-100.91	-24.49	-5.94	-1.48	-0.37	-133.19

The table 2.1 above shows that, given a 1.8% probability that an arriving customer will meet $k \geq 4$ customers in the system, the system discriminative index: $E\left[\widetilde{D}_{(k)}\right] = -133.19 < 0 \ \approx -133$, thus, a high discriminative system.

4.3.2. System Unfairness Coefficient:

Let $E[D^2|k]$ for $k = 0,1,2, \ldots$ denote the expected value of the square of the system discrimination, given that an arriving customer encounters k customers in the system. Let P_k denote the steady state probability that there are $k$customers in the system; then by equation $(3.0.4)$ the weekly unfairness index of the system, given that an arriving customer meets $k \leq 4$ customers in the system is given by table 3.0 below:

Table 3.0: Unfairness With Respect to $P_{k\leq 4}$ Customers in the System

No. of Customers:	1 Customer	2 Customers	3 Customers	4 Customers	$P_{k\leq 4}$
Prob of k-Customers:	0.3672	0.1765	0.0565	0.0136	0.6138
$E[D^2|k]P_k = D^2P_k$:	202,167.1	97.174.55	31,106.87	7487.67	240,859.36
$E[D|k]P_k = DP_k$:	-272.46	-130.96	-41.92	-10.09	-455.43

By equation (3.0.8), the overall weekly unfairness, given that an arriving customer meets $k \leq 4$ customers in the system is:

$$Var[D] = E[D^2] - \left[E[D]\right]^2 = 240{,}859.36 - [-455.43]^2 = 33{,}442.87\ (3.1.2)$$

The table 3.0 above shows that, given 61.38% probability that an arriving customer will meet$k \leq 4$ customers on a less busy days, the system unfair coefficient$Var[D] = 33{,}442.85$; thus, highly unfair system. Similarly, the weekly discrimination (unfairness) of the system, given that an arriving customer meets $k \geq 4$ customersin the system is given by table 3.1 below:

Table 3.1: Unfairness With Respect to $P_{k\geq 4}$ Customer in System

No. of Customer:	4 Cust.	5 Cust.	6 Cust.	7 Cust.	8 Cust.	$P_{k\geq 4}$
Prob of k-Cust:	0.0136	0.0033	0.0008	0.0002	0.00005	0.018
$E[D^2\|k]P_k = D^2P_k$:	7487.67	1816.86	440.45	110.11	27.53	9882.62
$E[D\|k]P_k = DP_k$:	- 10.091	-2.45	-0.594	-0.148	-0.037	-13.32

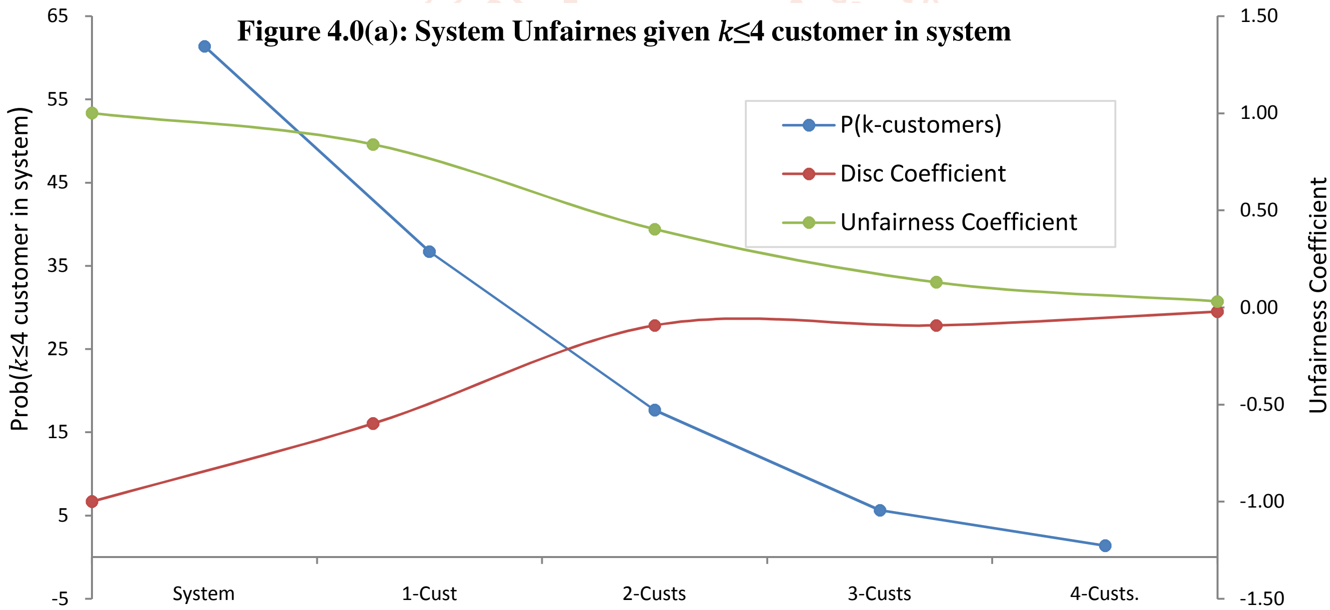
Similarly, by equation (3.0.8), the overall unfairness of system given that an arriving customer meets $k \geq 4$ customerson the queue:

$Var[D] = 9{,}882.62 - [-13.32]^2 = 9{,}705.2$

The table 3.1 above shows that, given a 1.8% probability that an arriving customer will meet$k \geq 4$ customers in the queue on less busy days, the system unfairness coefficient: $Var[D] = 9{,}770.51$; alsoa highly unfair system. Finally, the confidence interval (CI) for the validity/reliability of the unfairness coefficient is given by:

$CI = D \pm t_{\alpha/2}\sqrt{Var[D]}$, where$\alpha = 0.05$ (5%) 3.1.3

For $P_{k\leq 4}$ $customer$ in the System:$CI = -742 \pm (6.314)182.87 = (-1{,}896.641, 412.64)$, and for $P_{k\geq 4}$ $customers$ in the System:$CI = -742 \pm (6.314)98.85 = (-117.8611, -1{,}366.14)$.The figures 4.0(a)&(b) below, shows thatboth the discrimination index and the system unfairness coefficient increase with the probability of finding $k \leq 8$ customers on the queue. This result is intuitive and consistent with high negative discrimination index as well as high unfairness coefficient of the system which is synonymous with the deliberate violation of the service requirement preference principle inherent in queue fairness metrics.



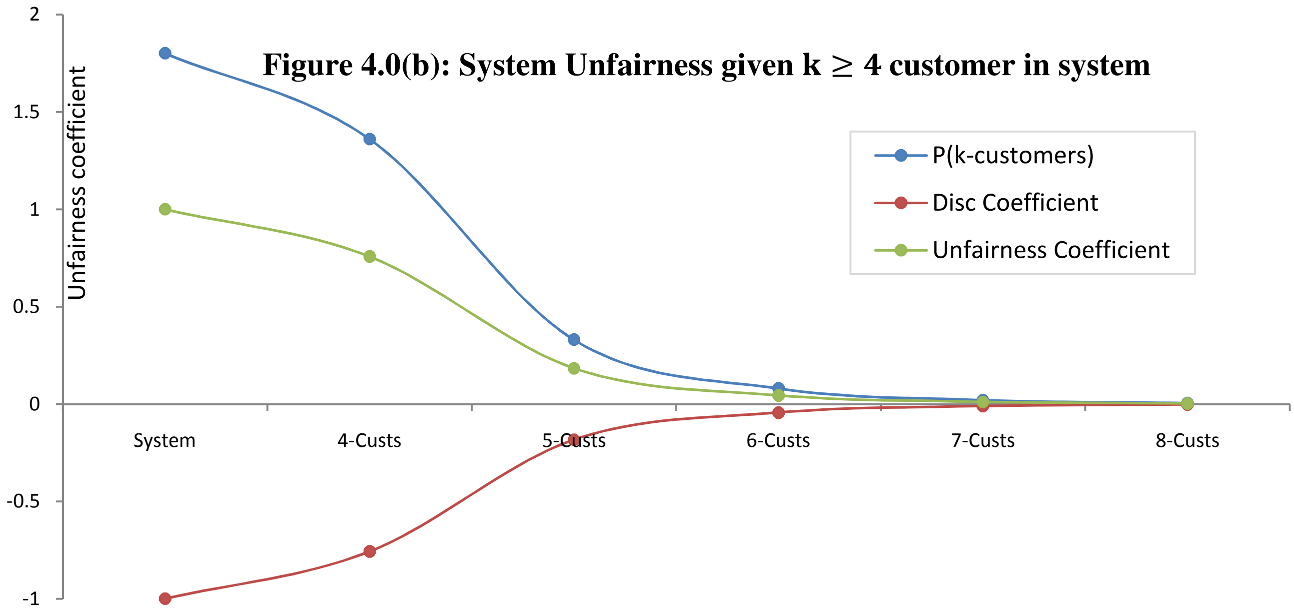


Figure 4.0(b): System Unfairness given k ≥ 4 customer in system

5. SUMMARY OF THE ANALYSES

From the analysis so far, the summarized performance and unfairness characteristics below are typically necessary for drawing a valid conclusion and recommendations on the ATM queuing systems under consideration. Tentatively, table 5.0 summarizes the operational characteristic while table 5.1 represents a summary of the unfairness characteristics of the System respectively.

Table 5.0: Operational Characteristic of the System

Characteristics	Values	Characteristics	Values
Mean arrival rate, (λ) Customers/hr	10.00	Probability that a customer balk or renege on busy days, %	1.8
Mean service rate, (μ) Customers/hr	10.00		
Traffic Intensity, (ρ) %	24.03	Probability that a customer enters system on busy days, %	98.2
System utilization, %	75.97		
Probability of empty system, (P_0) %	38.2	Delay Probability for less busy days, %	8.96
Probability busy server, $(1 - P_0)$ %	61.8	Delay Probability for busy days %	91.04
Probability that a customer enters system on less busy days, %	61.38	Mean Queue Length (L_q), customers	8.0
		Mean No of customers in system (L_s)	9.0
Probability that a customer balk or renege on less busy days, %	38.62	Mean waiting time on queue (W_q) hrs	0.8
		Mean Service time in system (W_s) hrs	0.9

Table 5.1Unfairness Characteristics of the system

Characteristics	Values
Total warranted service rate, $\omega(t)$ customers per day	560.00
Momentary warranted rate, $R(t)$ customers per server per day	140.00
Momentary granted rate, $\sigma(t)$ customers per server per day	34.00
Momentary discrimination, $\delta(t)$ customers per server per day	-106.00
Accumulative discrimination, $D(t)$ customers per server per week	-742.00
System discrimination when $k \leq 4$; $E\left[\widetilde{D}_{(k)}\right]$, customers per week	-4,554.00
System discrimination when $k \geq 4$; $E\left[\widetilde{D}_{(k)}\right]$, customers per week	-133.00
System Unfairness Coefficient when $k \leq 4$; $Var[D]$,	33,442.87
System Unfairness Coefficient when $k \geq 4$; $Var[D]$,	9,705.20

5.2. Discussions

Given the dual objectives of the study, our discussions on the results of the analyses of the ATM queuing system will focus on two major sub-headings: the implications of the system operating characteristics as exhibited by table 5.0, as well as that of the system unfairness characteristics as represent by table 5.1. These will elicit the relevant conclusions and recommendations necessary for the enhancement of service delivery in the institution under consideration as well as in other related organizations.

5.2.1. Implications of the System Operating Characteristics:

Considering the results summary on table 5.0 above, it is clear that the ATM service rate is greater than the customers' arrival rate; hence the system utilization factor being less than one (1) which connotes a stable queue. Similarly, the system traffic intensity indicates an average granted service rate of 24.03% customers per busy ATM per hour, while the utilization factor indicates an average of 75.97% customers on queue per hour; an evidence of slow or overutilization of servers. Furthermore, if the additional time required for a customer to complete his/her services is exponentially distributed with meanρ^{-1}, then the result of the analyses shows that a tagged customer must sojourn on the ATM terminal for at least4.16 minutes before departure on less busy days. Given each ATM serving rate of 1.5 minutes per customer, this lengthy sojourn time is intuitive and may be synonymous with the slow processing speed as well as the epileptic or poor internet connectivity of most bank ATM facilities in Nigeria.

The table 5.0 also shows that the probability of an arriving customer waiting on the queue before entering into service on less busy days is 8.96%; while on the busy day, an arriving customer wait for 91.04% of the time before entering into service. This observation which is intuitive and consistent with normal queuing system, may not be unconnected with upsurge of customers on the peak days (Monday, Saturdays and Sunday), while the less busy days are defined by low customers' traffic. This high probability of waiting on the busy days is sufficient to cause high customers' dissatisfaction rate and hence balk or renege from the system. The results also show that on busy days, an arriving customer only balked or reneged at 1.8% of the times, but enters the system at 98.2% of the times. This observation is a misnomer, but consistent with the system violation of the service requirement preference principle inherent in queuing systems.

However, given that the four ATMs capacity is 4 customers at any epoch, then a 61.38% probability that an arriving customer will enter the system on less busy days or balk or renege seems to be consistent with conventional customers' behaviour. Finally, for a system that has the capacity to work for 61.8% of the time per day and serving 10 customers per ATM per hour; representing an average of 40 customers per hour and 560 customers per day, yet granting services to only 135 per day is also a misnomer. This is insinuate huge number of customers' balking or renege, if the element of ATM malfunctioning, epileptic or poor internet connectivity is negligible. On the waiting time threshold, the results of the analyses also show that a tagged customer spent at least 48 minutes on the queue before entering into service, but spent about 6 minutes on the ATM terminal. This observation is also a misnomer though insinuates customers' literacy level in ATM operations as well as element of ATM malfunctioning, epileptic electricity supply or poor internet connectivity. Tentatively, this also accounts for the 91.04% delay probability of the system on the busy days as well as the wide difference in customer's sojourn time and the time a customer spent on the ATM terminal; as the system treats all customers with varying job size as a single class of customer.

5.2.2. Implications of the System Unfairness Characteristics:

Since a system serving a queue of people is a microcosm social construct, therefore, its fairness performance metric must conform to the three granular levels of the social system namely: (i) the customers' discrimination level (ii) the scenario unfairness level and (iii) the general system unfairness levels. On the customer's discrimination level, the summary of the analyses in table 5.1 shows that the performance metric of the Banks queuing system under study exhibits a high customer discrimination value; as the momentary discrimination index$\delta(t)$ indicates a negative value of $-106 < 0$.This index implies that at least a total of 106 customers are dissatisfied per day with the ATM service delivery system, and hence balk or renege. On the scenario unfairness level of the system, which connotes a summary statistic of the discrimination experienced by a (finite or infinite) set of jobs in a particular scenario (a sample path), the cumulative discrimination (under service) of the system over a week operations$D(t)$, also exhibit a negative value of $-742 < 0$. This is implies that at least a total of 742 customers per week or 2,968 customers per month are dissatisfied with the ATM service delivery system and hence balk or renege.

Finally, on the probability of the system discriminating customers on the less busy days, the results of the analyses also indicate a negative system discriminative value of$-4{,}554 < 0$, while system discriminative value for busy days is$-133 < 0$. This implies that because of the erratic modus operandi of the system, more customers are discriminated or dissatisfied on less busy days than the busy days. Corroborating these observations, the general system unfairness coefficient for both the less busy and busy days of the system, also exhibited high unfair coefficients of 33,442.85 and 9,770.51 respectively. In summarily, the figures 4.0(a) and 4.0(b) show that, both the unfairness and customer discrimination of the system in general increase with the probability of having $k > m$customers on the queue and vice versa. Tentatively, both the high negative discrimination index and unfairness coefficient of the system as well as their variations with the probability of having $k > m$ customers on the queue may not be unconnected with the system violation of the service requirement preference principle inherent in queue fairness metrics; as the system treats all customers with varying job sizes as a single class of customers.

5.3. Conclusion

The study is an analysis of the performance measures as well as the unfairness characteristics of the multi-server single-queuing system of some financial institutions in Kaduna Nigeria, using exponential inter-arrival/service time distribution and RAQF metrics respectively. The result of the analyses summarily shows that the high delay probability, high negative discrimination index as well as the high system unfairness coefficient which varies with the probability of having customers on the queue on the peak period may not be unconnected with the system violation of one of the fundamental principles of queue fairness – service requirement preference principle. Therefore in the ATM queuing systems, if service order is based only on customer arrival time (job seniority), with no consideration given to service times (service requirement), there is a high probability that customers with short jobs may wait till eternity for the execution of long jobs and thus, negatively discriminated. The results derived in this work can serve for two purposes. First, the simpler results, which might sound intuitive to many researchers, can be used to build confidence both in the RAQF and customer discrimination metrics; both of which are very new to the queuing theory and require examination and trust building. Second, once the confidence is built, the metrics can be used to study and evaluate systems where the results may not be explicit.

Finally, notwithstanding, its immense contribution to the growing literature on queue model applications and OR discipline, the kernel of our model and its analytical results/findings will help to elicit policy formulation and research on queue fairness as an indicator for customers' satisfaction and service delivery quality in service industries. This will guarantee possible system modification and resources optimization as well as strengthening the socio-economic and developmental indicators for monitoring, benchmarking, evaluation, forecasting and overall planning in the service delivery sector of the Nigerian economy. The evolving partnership between service delivery industries and academia will foster innovations and system interventions in line with international best practices. Besides providing a robust problem-solving technique for the real-world situation with respect to optimal deployment of ATM technology, the result of the analyses will elicit the necessary information for optimizing customers' satisfaction in the general business sector.

5.4. Recommendations

From the results of our analyses, it is recommended that:

1. Since all customers arriving at the system at any epoch have varying service requirement (job size), then classification of customers based on service requirement, and or dedicating separate ATM machines to similar job-size queue will increase the fairness and hence reduce customer's dissatisfaction index of the system.
2. To reduce the ATM overutilization factor as well as customer's delay probability associated with the system, the deployment of few ATMs with optimal processing speed would be more cost effective than more ATMs with relatively slow processing speed.
3. Since the effective operation of the ATM is a function of the quality of internet connectivity, electricity supply and customer's computer literacy, therefore, the employment of solar electricity, stable internet connectivity as well as basic ATM operation orientation to customers would reduce customer's sojourn time in the service point, increase fairness index and hence guarantee customer's satisfaction.
4. That the RAQF unfairness metric should be used in evaluating the fairness aspects of alternative queue systems that may be proposed in future.

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