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Some Results in Ternary Lateral Sub Idempotent

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ABSTRACT

Some special elements such as ternary lateral sub idempotent, regular, singular were characterized by various properties in ternary semiring. We have examined the equivalent conditions like the relation between binary regular, ternary regular and also Regular, Right regular, Left regular and Lateral regular etc. We proved that (Left, Right, Lateral) singular elements are multiplicative idempotent undermost some special condition.

KEYWORDS: *Idempotent elements, Regular, Singular, Ternary semiring (TSR)*

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1. INTRODUCTION

A Ternary semiring is an algebraic system with operations binary addition and ternary multiplication. The theory of algebraic system was introduced by Lehmer [7]. He investigated certain ternary algebraic systems called triplexes which turn out to be commutative ternary groups. Madhusudhana Rao and srinivasa Rao [3] characterized the ternary semiring. Ternary semiring has introduced in [6] and also studied the properties. Vasanthi and Sulochana looked into the fundamental structure of sub idempotent semirings [1,2,4]. Some properties of idempotent semirings are discussed in [5]. Our main purpose of this paper is to study the ternary lateral sub-idempotent of ternary semiring.

we characterize some special equivalent class of ternary lateral sub idempotent in ternary semiring. We discuss the various structure of ternary lateral sub idempotent in ternary semiring. The main objective of this article ternary lateral sub idempotent element in T and their properties using additive and multiplicative cancellation laws.

2. Preliminaries

Definition 2.1. A non-empty set T is an additive commutative semigroup that satisfies the following requirements, it can be considered a TSR. This is achieved by combining a binary operation called addition with a Ternary multiplication symbolized by juxtaposition

(i) $abc \in T$,
(ii) $[abc]de = a[bcd]e = ab[cde]$,
(iii) $[a+b]cd = acd + bcd$,
(iv) $a[b+c]d = abd + acd$,
(v) $ab[c+d] = abc + abd$ for all $a,b,c,d,e \in T$

Example 2.1. Let the set of all negative integers be Z^{-}. subsequently combining Ternary multiplication and binary addition [ ] defined by

$[abc] = abc$ for all $a,b,c \in Z^{-}$, forms a T.S.R.

Definition 2.2. A Ternary semiring T is said to be multiplicatively left cancellative (MLC), (laterlly cancellative (MLLC) and right cancellative (MRC) if $abx = aby$, $(axb = ayb$ and $xab = yab)$ implies that $x = y$, for all $a,b,x,y \in T$

Definition 2.3. For any elements a and b in T, satisfies $a + b = a(a + b = b)$, subsequently a Ternary Semigroup $(T, +)$ becomes a left (right) singular.

Definition 2.4. For any element 'a' in a Ternary semiring T,

$a^3 = a\,(a + a = a)$ indicates that it is idempotent (additively idempotent)

Definition 2.5. If $a + a^3 = a$, then the member 'a' in T is regarded to be MSI.

Definition 2.6. If $ab^2 = a\,(b^2 a = a, bab = a)$ for all $a, b \in T$, then a Ternary semiring $(T, .)$ is considered left (right, lateral) singular. If a ternary semiring $(T, .)$ is left, right, and lateral singular, it is referred to as singular.

Definition 2.7. An element 'a' in TSR is defined as regular(additive regular) if there exist $b \in T \ni a = ababa, b \in a\{1\}\,(a = a + b + a,$ for some b is additive 1-inverse of a). The TSR T is said to be regular if every element in T is regular.

Definition2.8. For $a \in T$ such that $a = a^3xy\ \ (a = xya^3, a = xa^3y)$ for all $x, y \in T$, then a Ternary semiring $(T, .)$ is considered left (right, lateral) regular. If a ternary semiring $(T, .)$ is left, right, and lateral regular, it is referred to as regular.

3. Results and Discussion

In this section we have generalized the Definition(5.1.1) [8, p.78] for ternary semiring and obtained some results.

Definition 3.1 An element $a \in T$ is said to be ternary left sub idempotent if $a + a^3 + ba^2 = a$ for some $b \in T$.

Definition 3.2 An element $a \in T$ is said to be ternary lateral sub idempotent if $a + a^3 + aba = a$ for some $b \in T$.

Definition 3.3 An element $a \in T$ is said to be ternary right sub idempotent if $a + a^3 + a^2b = a$ for some $b \in T$.

Definition 3.4 A ternary semiring $(T, .)$ is said to be ternary two sided sub idempotent, if it is both ternay left and right sub idempotent.

Definition 3.5 A ternary semiring $(T, .)$ is said to be ternary sub idempotent, if it is both ternary left, lateral and right sub idempotent.

Lemma 3.1 Let $(T, +, .)$ be a ternary lateral sub idempotent in ternary semiring then $(T, +)$ is regular

Proof : Since 'a' is ternary lateral sub idempotent we have $a + a^3 + aba = a$. Using additive left and right cancellation law we obtain $a + b + a = a$. Therefore T is additive regular.

Theorem 3.1 Let $(T, +, .)$ be a ternary lateral sub idempotent in ternary semiring then the following are equivalent

(i) $(T, +)$ is left singular

(ii) $(T, .)$ is multiplicative sub idempotent

(iii) $(T, +)$ is idempotent

Proof : (i)⇒(ii)

Since 'a' is additive left singular and ternary lateral sub idempotent we have $a + a^3 + aba + b = a + b$. Using the additive right cancellation law which implies $a + a(a + b)a = a$. In the above equation useing (i) we obtain $a + a^3 = a$. Therefore a ternary semiring $(T, .)$ is multiplicative sub idempotent.

(ii)⇒(iii)

Since 'a' is multiplicative sub idempotent we have $a + a^3 = a$ which implies that $a + a^3 + a^3 = a + a^3 \Rightarrow a(a + a)a = a^3$ we obtain $a + a = a$. Therefore a ternary semiring $(T, +)$ is idempotent.

(iii)⇒(i)

Since 'a' is additive idempotent and ternary lateral sub idempotent, from Lemma (3.1) we have $a + a = a + b + a$. Using the additive right cancellation law which implies $a = a + b$ we obtain $a + b = a$. Therefore a ternary semiring $(T, +)$ is left singular.

Corollary 3.1 Let $(T, +, .)$ be a ternary lateral sub idempotent in ternary semiring then the following are equivalent

(i) $(T, +)$ is right singular

(ii) $(T, .)$ is multiplicative sub idempotent

(iii) $(T, +)$ is idempotent

Theorem 3.2 Let $(T, +, .)$ be a ternary lateral sub idempotent if $(T, .)$ is idempotent then $(T, +)$ is commutative.

Proof : Since 'a' is idempotent and ternary lateral sub idempotent we have $a^3 = a + a^3 + aba$

$a^3 + aba + a^3 = a + a^3 + aba$

$a + aba + a^3 = a + a^3 + aba$

$aba + a^3 = a^3 + aba$

$a(b + a)a = a(a + b)a$

$(b + a) = (a + b)$

Therefore (T, +) is commutative.

Theorem 3.3 Let $(T,+,.)$ be a ternary lateral sub idempotent and additive left singular then $a+a^{2n+1}=a$ for all $a\in T$, $n\in\mathbb{N}$

Proof : Since 'a' is additive left singular and ternary lateral sub idempotent then using Theorem (3.1) we have $a+a^3=a$ which implies that $a+a.a^2=a$

$a+(a+a^3).a^2=a$

$a+a^3+a^5=a$

$a+a^5=a$ -----------------------------------(3.1)

$a+a^5+a^7=a$

$a+a^7=a$ -----------------------------------(3.2)

In general Equation (3.1), (3.2) we have the following $a+a^{2n+1}=a\ \forall a\in T, n\in\mathbb{N}$

Theorem 3.4 Let $(T,+,.)$ be a ternary lateral sub idempotent if $(T,.)$ is multiplicative sub idempotent then $(T,.)$ is idempotent.

Proof : Since 'a' is multiplicative sub idempotent and ternary lateral sub idempotent we have $a+a^3=a+b+a$ (from Lemma (3.1))

$a+b+a^3=a+b+a$ (from Theorem (3.1))

$a^3=a$

Therefore $(T,.)$ is idempotent.

The following theorem deals the relation between binary regular and ternary regular.

Theorem 3.5 Let $(T,+,.)$ be a ternary lateral sub idempotent and $(T,+)$ is right singular then the following are equivalent

(i) $ababa=a$

(ii) $aba=a$

Proof : (i)⇒(ii)

Let $ababa=a$

$(a+b+a)baba=a+a^3+aba$

$ababa+b^2aba+ababa=a+a^3+aba$

$b^2aba+ababa=a(a+b)a$

$b^2aba+ababa=aba$

$(b^2a+aba)ba=aba$

$b^2a+aba=a$

$(b+a)ba=ababa$

$b+a=aba$

$a=aba$

(ii)⇒ (i) which is obvious.

Corollary 3.2 Let $(T,+,.)$ be a ternary lateral sub idempotent and $(T,+)$ is right singular then the following are equivalent

(i) $ababa=a$

(ii) $aba=a$

Theorem 3.6 Let $(T,+,.)$ be a ternary lateral sub idempotent then the following are equivalent

(i) $baa=a$

(ii) $aba=a$

(iii) $aab=a$

Proof : (i)⇒(ii)

Since 'T' is ternary lateral sub idempotent and $baa=a$ we have $b(a+a^3+aba)a=a$

$baa+ba^4+babaa=a$

$a+a^3+babaa=a$ -------------------------(3.3)

$a+a^3+a=a+b+a$ (from Lemma (3.1))

$a^3=b$ -----------------------------(3.4)

Equation (3.4) in (3.3) we get $a+b+babaa=a+b+a$

$babaa=baa$

$aba=a$

From Equation (3.4) we get $baaaa=b$ which implies that $bab=b$

Therefore $aba=a$ and $bab=b$

(ii)⇒(iii)

Let $aba=a$

$ab(a+a^3+aba)=a$

$aba+aba^3+ababa=a$

$a+aba^3+a=a+a^3+a$

$abaaa=a^3$

$aaaba=aaa$

Therefore $aab=a$

(iii)⇒(i)

Let $aab=a$

$a(a+a^3+aba)b=a$

$aab+a^4b+aabab=a+a^3+aba$

$aabab=aba$

Therefore $a=baa$

Theorem 3.7 Let $(T,+,.)$ be a ternary lateral sub idempotent then the following are equivalent

(i) $(T,.)$ is idempotent

(ii) $(T,.)$ is Lateral singular

(iii) $(T,.)$ is Right singular

(iv) $(T,.)$ is Left singular

Proof : (i)⇔(ii)

Since 'a' is idempotent and ternary lateral sub idempotent $a(a+b+a)a = a$ (from Lemma (3.1)) which implies that $a^3 + aba + a^3 = a \Rightarrow a + aba + a = a + b + a \Rightarrow aba = b$.Therefore $(T,.)$ is Lateral singular in ternary semiring.

Conversly, since $(T,.)$ is lateral singular and ternary lateral sub idempotent we have $a(b+a+b)a = b$ which implies that $aba + a^3 + aba = b + a + b$ $\Rightarrow a^3 = a$. Therefore $(T,.)$ is idempotent.

Similarly we can prove (i)⇔(iii) and (i)⇔(iv)

Theorem 3.8 Let $(T,+,.)$ be a ternary lateral sub idempotent then the following are equivalent
(i) $(T,.)$ is regular
(ii) $(T,.)$ is Right regular
(iii) $(T,.)$ is Left regular
(iv) $(T,.)$ is Lateral regular

Proof : Since Lemma (3.1) we have $a+b+a = a$ and $b+a+b = b$

(i)⇒(ii)

Let a ternary semiring is regular and ternary lateral sub idempotent we have $(a+b+a)baba = a$

$a + bbaba + a = a + b + a$

$bbaba = b$ --------------------------(3.5)

$bba(b+a+b)a = b$

$b + bbaaa + b = b$

$bba^3 = a$ (from Equation (3.5))

Therefore $(T,.)$ is right regular.

(ii)⇒(iii)

Let a ternary semiring is right regular and ternary lateral sub idempotent we have

$bb(a+b+a)aa = a$

$a + bbbaa + a = a + b + a$

$bbbaa = b$ --------------------------(3.6)

Since T is right regular we have $bbaaa = a$

$bbaabbaaa = bbaaa$

$baabbaa = baa$

$b^3aaaabbaa = b^3aaaa$ (from Equation (3.6))

$aaabbaa = aaa$

$a^3bb = a$

Therefore $(T,.)$ is left regular

(iii)⇒(iv)

Let a ternary semiring is left regular and ternary lateral sub idempotent we have $(a + b + a)aabb = a$

$a + ba^2b^2 + a = a + b + a$

$ba^2b^2 = b$ --------------------------(3.7)

$baa(b+a+b)b = b$

$b + ba^3b + b = b$ (from Equation (3.7))

$ba^3b = a$

Therefore $(T,.)$ is lateral regular.

(iii)⇒(iv)

Let a ternary semiring is latera regular and ternary lateral sub idempotent we have $ba(a + b + a)ab = a$

$a + babab + a = a + b + a$

$babab = b$ --------------------------(3.8)

Since T is lateral regular we have $ba^3b = a$

$bababaaab = ba^3b$ (from Equation (3.8))

$ababaaa = a^3$

$ababa = a$

Therefore $(T,.)$ is regular.

Theorem 3.9 Let $(T,+,.)$ be a ternary lateral sub idempotent and regular then $a^{4n+1} = a$ for all $a,b \in T$ and $n \in \mathbb{N}$

Proof : Since 'a' is regular and ternary lateral sub idempotent we have $a(b+a+b)aba = a$ which implies that $a + a^3ba + a = a + b + a$

$a^3ba = b$ --------------------------------(3.9)

$aaa(b + a + b).a = b$

$a^3ba + a^5 + a^3ba = b$

$b + a^5 + b = b$ (from Equation (3.9))

$a^5 = a$ ----------------------------------(3.10)

$a^5.a^4 = a$

$a^9 = a$ ----------------------------------(3.11)

In general Equation (3.10), (3.11) we have the following $a^{4n+1} = a \ \forall a \in T, n \in \mathbb{N}$

Remark 3.2 In the above theorem also we can prove for right regular, left regular and lateral regular.

Lemma 3.2 Let $(T,+,.)$ be a ternary lateral sub idempotent and $aba = a$ then $a^3 = b$

Proof : Since 'a' is ternary lateral sub idempotent and $aba = a$ we have $a + a^3 + a = a$ which implies $a + a^3 + a = a + b + a$. Using additive left and right cancellation law we obtain $a^3 = b$.

Theorem 3.10 Let $(T, +, .)$ be a ternary lateral sub idempotent and $aba = a$ then $a^{4n-1} = b$ for some $a, b \in T$ and $n \in \mathbb{N}$

Proof : Since T is ternary lateral sub idempotent and Lemma (3.2) we have

$a^3 = b$ ---------------------------------(3.12)

$(a + a^3 + aba)a^2 = b$

$a^3 + a^5 + aba^3 = b$

$a^3 + a^5 + a^7 = a^3$ (from Lemma (3.2))

$a + a^3 + a^5 = a$

$a + b + a^5 = a + b + a$ (from Lemma (3.1) and (3.2))

$a^5 = a$ -----------------------------(3.13)

From the Equation (3.13) in (3.12) we get $a^5 . a^2 = b$ which implies that

$a^7 = b$ ------------------------------(3.14)

From the Equation (3.14) in (3.12) we get $a^5 . a^6 = b$ which implies that

$a^{11} = b$ ------------------------------(3.15)

Equations (3.12), (3.14), (3.15) continueing in this way in general, we have the following $a^{4n-1} = b$ for some $a, b \in T$ and $n \in \mathbb{N}$

Remark 3.1 In theorem (3.9) and (3.10) deals ternary lateral sub idempotent with ternary regular and binary regular, if TSI and ternary regular we have $a^{4n+1} = a$ likewise ternary lateral sub idempotent and binary regular we have $a^{4n-1} = b$

Theorem 3.11 Let $(T, +, .)$ be a ternary lateral sub idempotent and lateral regular then $a^{4n}b = b$ for some $a, b \in T$ and $n \in \mathbb{N}$

Proof : Since T is ternary lateral sub idempotent and lateral regular we have

$(b + a + b)a^3b = a$ (from Lemma (3.1))

$ba^3b + a^4b + ba^3b = a$

$a + a^4b + a = a + b + a$

$a^4b = b$ ------------------------------(3.16)

Since the Equation (3.16) we get $a^4 . a^4b = b$ which implies that

$a^8b = b$ ------------------------------(3.17)

From the Equation (3.17) we get $a^8 . a^4b = b$ which implies that

$a^{12}b = b$ ------------------------------(3.18)

Equations (3.16), (3.17), (3.18) continuing in this way in general, we have the following $a^{4n}b = b$ for some $a, b \in T$ and $n \in \mathbb{N}$

References

[1] Sulochana. N and Vasanthi. T, Structure of some Idempotent Semirings, Journal of Computer and Mathematical Sciences, Vol 7(5), 294-301, (May 2016).

[2] Vasanthi. T and Sulochana. N, Semirings Satisfying the Identities, International Journal od Mathematical Archive – 3(9), 3393-3399 Sept. – (2012).

[3] Madhusudhana Rao. D and Srinivasa Rao. G, Characteristics of Ternary Semirings, International Journal of Engineering Research and Management, Vol.2, No.1, January 2015.

[4] Vasanthi. T, Monikarchana. Y and Manjula. K, Structure of Semirings, Southeast Asian Bulletin of Mathematics, Vol.35, (2011), PP.149-156.

[5] Amala. M and Vasanthi. T, Idempotent Property of Semirings, International Research Journal of Pure Algebra, 5(9), (2015), 156-159.

[6] Dutta. T.K and Kar. S, On regular ternary semirings, Advances in Algebra, Proceedings of the ICM Satellite Conference in Algebra and Related Topics, World Scientific, New Jersey, (2003), 343-355.

[7] Lehmer. D.H, A Ternary Analogue of Abelian Groups, American Journal of Mathematics, 59 (1932), 329-338.

[8] Madhusudana Rao. D and Srinivasa Rao. G, A study on Ternary Semirings, International Journal of Mathematical Archive, 5(12), (2014), 24-30.

[9] Deepan. B and Anbalagan. S, Review of Additive singular in Ternary Semiring, Indian Journal of Natural Sciences, Vol.15/Issue 85/Aug 2024, 76459-76467.

[10] Mallikarjuna. G, On some studies of Semirings and Ternary Semirings, Sodhganga, thesis January 2017, pp 78-86.

[11] Deepan. B and Anbalagan. S, Review on Natural Idempotent Pair in Ternary Semiring, Indian Journal of Natural Sciences, Vol.15/Issue 87/Dec 2024, 76459-76467.